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Day-Ahead Load Forecasting Using Exponential Smoothing

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Abstract

The existence of a liberated market for electricity implies the necessity of using load forecasting in order to optimize and reduce the costs for the electric energy consumption. Different strategies for the purchase of electrical energy require precise load forecasting. The Day Ahead Market makes possible the acquisition of electricity for one day in advance. This reduces the risk of making electrical energy transactions on Balance Market, which means buying at a very high price and selling at a very low price.

This paper presents a Day Ahead Load Forecasting approach for an industrial costumer using exponential smoothing method. The purpose of this paper is to present a new method for day ahead load forecasting.

Keywords: Load forecasting, liberated market, electrical energy consumption, day ahead market, exponential smoothing

1. Introduction

The day-ahead load forecasting can have a significant technical impact on the grid assuring the economic energy dispatch, the optimal unit commitment, the security assessment and reducing power losses. Due to its importance from economical point of view and to the high complexity of electrical systems, the short-term forecast (from several hours to several days) has been a continuing concern of professionals in order to improve its performances. For this purpose, numerous techniques and methods of approach have been used; they include: linear models and/or non-linear time series, regression techniques, model expert systems, fuzzy and neural networks approaches, etc. The exponential smoothing belongs to time series methods.

Time series data often arise when monitoring industrial processes or tracking corporate business metrics. The essential difference between modeling data via time series methods or using the process monitoring methods is that the data points taken over time may have an internal structure (such as autocorrelation, trend or seasonal variation) that should be accounted for [2]. This paper will present a day-ahead load forecasting for a fittings manufacturer for a period of one week using Holt and Winters Exponential Smoothing method [1].

2. Exponential Smoothing method

The method used for the load forecast is based on time series and takes into consideration only the history of the consumption in order to establish a pattern in the past that might be useful and similar with the present load curves. This technique uses exponentially decreasing weights as the observation get older. Recent observations are given relatively more weight in forecasting than the older observations. Exponential Smoothing is used to generate the smoothed values in order to obtain estimates [1].

Exponential smoothing types currently used are:

- the first order exponential smoothing;
- the second order exponential smoothing;
- the higher order exponential smoothing;
- the Holt-Winters mechanism.

2.1 The first-order exponential smoothing

It uses a recursive equation that can also be seen as the linear combination of the current observation and

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smoothed observation of the previous time unit. As the latter contains the data from all previous observations, the smoothed observation at moment T (estimated time) is in fact the linear combination of the current observation and the discounted sum of all previous observations:

$$\tilde{y}_{T+1} = \lambda \cdot y_T + (1-\lambda) \cdot \tilde{y}_T \quad (1)$$

where

$\tilde{y}_{T+1}$ is the estimated value for moment T+1;

y_T - the real value for moment T;

$\lambda \in [0,1]$ - discount factor.

The discount factor represents the weight put on the previous observation while $(1-\lambda)$ is the weight put on the smoothed value of the previous observations. The most important issue for the exponential smoothers is the choice of the discount factor, λ [1], [4].

2.2 The second-order exponential smoothing

The first-order exponential smoothing method was extended by Holt for presenting time series with trend (and random component). This approach adjusts the time series considering the trend to be linear:

$$\tilde{y}_{T+h} = a_t + h \cdot b_t \quad (2)$$

where

a_t - the value of the intercept;

b_t - the value of the slope;

h - time horizon.

For the one step ahead forecasting, the value of *h* is 1. Parameters a_t and b_t are calculated as follows:

$$a_t = \alpha \cdot y_t + (1-\alpha) \cdot (a_{t-1} + b_{t-1}) \quad (3)$$

$$b_t = \beta \cdot y_t + (1-\alpha) \cdot (a_t - a_{t-1}) + (1-\beta) \cdot b_{t-1} \quad (4)$$

where

$\alpha, \beta \in [0,1]$ are the discount factors (constants);

a_t, b_t - values for this parameters at time t;

a_{t-1}, b_{t-1} - values for this parameters at time t – 1.

The constants α, β will be chosen for the smallest sum of the squared forecast errors (the value of λ from the first-order exponential smoothing will be determined in same manner) [1], [3], [4].

$$e_t = \tilde{y}_t - y_t \quad (5)$$

$$SS_E = \sum_{t-1}^{T} e_t^2 \quad (6)$$

where

e_t - the forecast error;

$\tilde{y}_t$ - forecasted value;

y_t - real value;

SS_E - sum of squared forecast errors.

2.3 The higher-order exponential smoothing

The first and second order exponential smoothing can be extended to the general *n*-th degree polynomial model presented in the equation below:

$$y_t = \beta_0 + \beta_1 \cdot t + \frac{\beta_2}{2!} \cdot t^2 + \ldots + \frac{\beta_n}{n!} \cdot t^n + \varepsilon_t \quad (7)$$

where ε_t are assumed to be independent with mean 0 and constant variance σ_ε^2. In order to estimate the parameters the next equations will be used:

$$\begin{aligned} \tilde{y}_T^{(1)} &= \lambda \cdot y_T + (1-\lambda) \cdot \tilde{y}_{T-1}^{(1)} \\ \tilde{y}_T^{(2)} &= \lambda \cdot \tilde{y}_T^{(1)} + (1-\lambda) \cdot \tilde{y}_{T-1}^{(2)} \\ &. \\ &. \\ \tilde{y}_T^{(n)} &= \lambda \cdot \tilde{y}_T^{(n-1)} + (1-\lambda) \cdot \tilde{y}_{T-1}^{(n)} \end{aligned} \quad (8)$$

where $\tilde{y}_T^{(n)}$ is the estimated value for the *n*-th order exponential smoothing [1].

2.4 The Holt-Winters mechanism for seasonal time series [1]

Some time series data exhibit cyclical or seasonal patterns that cannot be effectively modeled using the polynomial model in equations (8). Several approaches are available for the analysis of this data. The methodology of the Day-Ahead forecast presented in this paper was introduced by Holt and Winters and is generally known as Winters' method [1]; in this case, a seasonal adjustment is made to the linear trend model. Two types of adjustments are used, namely the multiplicative and the additive model.

This paper uses a multiplicative model and the estimated values are calculated as in the following equation:

$$\tilde{y}_{T+h} = (a_t + h \cdot b_t) \cdot S_{t-p+h} \quad (9)$$

where

h - the time horizon;

$\tilde{y}_{T+h}$ - the forecasted value;

a_t - the level of the time series;

b_t - the trend of the time series;

S_{t-p+h} - the seasonal adjustment.

The parameters mentioned above are determined in the next recursive equations:

$$a_t = \alpha \cdot \left[\frac{y_t}{S_{t-p}} \right] + (1-\alpha)\cdot(a_{t-1} + b_{t-1}) \qquad (10)$$

$$b_t = \beta \cdot \left[\frac{a_t}{a_{t-1}} \right] + (1-\beta)\cdot b_{t-1} \qquad (11)$$

$$S_t = \delta \left[\frac{y_t}{a_t} \right] + (1-\delta)\cdot S_{t-p} \qquad (12)$$

where α, β, δ are the discount factors (constants) which must be chosen for the smallest sum of the squared forecast errors (equation 6), and p represents the period of the season. The discount can be determined also by the smallest MSE (mean square error – eq. 13), or by any statistical indicator taken into account (MAD, MAPE, MPE, ME) [1], [3], [4].

$$MSE = \frac{1}{n} \sum_{t-1}^{T} e_t^2 \qquad (13)$$

where n is the number of observations [1], [3].

3. Case study

A fittings manufacturer from Cluj-Napoca has been chosen as case study. The method used for the day-ahead load forecast is the Holt-Winters mechanism [1]. For the adjustment of the level, trend and seasonal indices, it was taken into account the interval from 01.11.2008 to 13.03.2009. The forecast has been done for 14.03.2009 – 20.03.2009 interval. The period of the cycle (p) taken into consideration for the day ahead forecast is one week (7 days). Each hourly interval is forecasted individually and the seasonal indices capture the type day of the week.

The initial values of the parameters (the level, the trend and the seasonal adjustment) were determined with the next equations [4]:

$$a_p = \frac{1}{p} \cdot \sum_{k=1}^{p} y_k$$

$$b_p = \frac{1}{p} \cdot \left(\frac{Y_{p+1} - Y_1}{p} + \frac{Y_{p+2} - Y_1}{p} + \ldots + \frac{Y_{2p} - Y_p}{p} \right)$$

$$S_1 = \frac{Y_1}{a_p}, S_2 = \frac{Y_2}{a_p}, \ldots, S_p = \frac{Y_p}{a_p} \qquad (14)$$

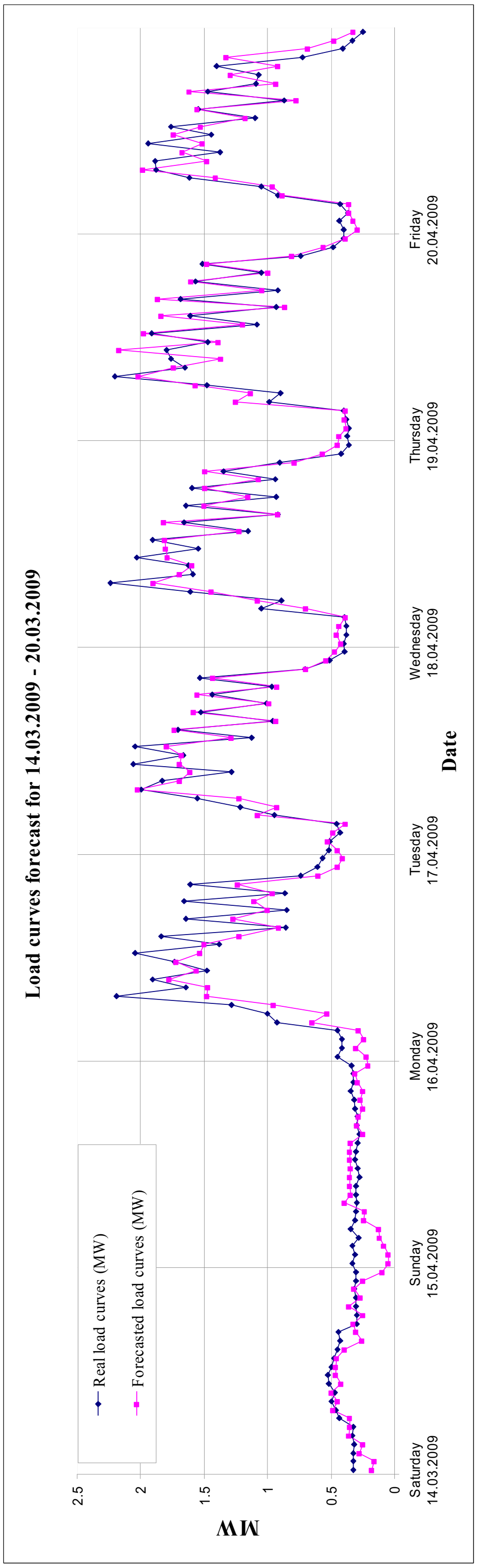

Figure 1. Load curves for the day-ahead load forecasting (14.03.2009 – 20.03.2009)

where a_p and b_p are the initial values for the level and trend of the first season. In the equations (9), the *p* is the period of the season (7 days).

α, β, δ discount factors were determined for the period 01.11.2008 – 20.03.2009 taking into consideration the smallest MSE, for each hourly interval; the discount factors values must be between 0 and 1. In order to establish the optimal values, software has been design to calculate the smallest MSE for the three indices. With an increment of 0.03 for each discount factor, starting from value 0 and ending close to 1, it has been determined for each hour the optimal values for α, β, δ (Table 1).

Table 1. α, β, δ and MSE optimal values for each hourly interval

Hour	1	2	3	4
MSE	0.0323	0.0260	0.0259	0.0214
α	0.57	0.51	0.39	0.48
β	0	0	0	0
δ	0.12	0.03	0.06	0.15
Hour	5	6	7	8
MSE	0.0373	0.0292	0.0594	0.0880
α	0.51	0.33	0.42	0.33
β	0	0	0	0
δ	0.03	0.09	0.09	0.45
Hour	9	10	11	12
MSE	0.0829	0.0829	0.1031	0.0803
α	0.27	0.3	0.42	0.33
β	0	0	0	0
δ	0.45	0.51	0.45	0.48
Hour	13	14	15	16
MSE	0.0929	0.0534	0.0416	0.0405
α	0.36	0.36	0.6	0.48
β	0	0	0	0
δ	0.36	0.36	0.12	0.09
Hour	17	18	19	20
MSE	0.0432	0.0270	0.0394	0.0278
α	0.48	0.48	0.54	0.51
β	0	0	0	0
δ	0.06	0.09	0.06	0.12
Hour	21	22	23	24
MSE	0.0415	0.0204	0.0217	0.0207
α	0.51	0.42	0.57	0.54
β	0	0	0	0
δ	0.09	0.21	0.39	0.12

For the forecasted period (14.03.2009 – 20.03.2009) the main statistical indicators which reveal the model performances are presented in Table 2.

Table 2. Statistical measures for the forecast period 14.03.2009 – 20.03.2009

	Exponential Smoothing				
	ME MW	MAD MW	MPE %	MAPE %	MSE MW^2
	0.0440	0.1405	5.34	10.13	0.0376
Max	0.7101	0.7101	86.23	86.23	
Min	-0.5988		-82.60		

The statistical measures presented in Table 2 are calculated as follows:

- Mean error: $ME = \frac{1}{n} \cdot \sum_{t=1}^{n} e_t$ (15)

- Mean absolute deviation: $MAD = \frac{1}{n} \cdot \sum_{t=1}^{n} |e_t|$ (16)

- Percent forecast error: $re_t = \left(\frac{y_t - \tilde{y}_t}{y_t} \right) \cdot 100$ (17)

- Mean percent forecast error: $MPE = \frac{1}{n} \cdot \sum_{t=1}^{n} re_t$ (18)

- Mean absolute percent forecast error:

$$MAPE = \frac{1}{n} \cdot \sum_{t=1}^{n} |re_t| \quad (19)$$

where *n* is the number of observations [1].

The load curves can be visualized in Figure 1 and show a poor forecast results for the hourly intervals 8 to 13 of each working day and much better in the rest hours of the day.

As mentioned in the paragraph before, Table 1 presents the values for the α, β, δ discount factors. It can easily be seen that the time series doesn't have a trend, because of the 0 values for the β index (Table 1). For some hourly intervals, δ is very low (close to 0), indicating the fact that the time series depends more on level of the series than on the seasonal adjustment. The smallest MSE on which the indices were chosen reveal greater values for the first working cycle of the day (8 to 14 hourly intervals), which can indicate that the time series for these hourly intervals are much hard to predicted than the other intervals.

4. Conclusions

Exponential smoothing is a statistical method of forecasting, seldom used for load forecasting due to poor results compared to the fitting techniques (linear regression, fuzzy, neural networks etc.). However, if the time series is stationary and the consumption is similar to the recent past, without any important variance in time, it might be useful to use a simpler technique for the load forecast than a sophisticated method that could introduce easily errors.

The model proposed in this paper for the load forecast of an industrial costumer (fittings manufacturer) is one that is not adaptable to great changes. The statistical measures indicate good results for the MAD (Table 2) to a very low value, approximately 7% of the maximum load for the forecasted period. MSE value indicates good results and low load forecast error. Unfortunately, the MAPE is too large compared to those required by an accurate forecast. The maximum and minimum absolute error, as the actual error, indicates a problem with the chosen model for some of the hourly intervals.

Future research will be initiated in order to separate holydays from the normal days and use appropriated time series for the load forecast of each day type.

The method can be adjusted and probably improved using fitting techniques together with exponential smoothing for a more accurate forecast.

5. References

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