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Focusing and Deflecting Systems Modelling and Simulation of the Electron Beam Equipment CTW 5/60

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Abstract

Electron beam material processing is an important nonconventional technique used in many industrial fields where the conventional methods proved to be inefficient. The entire process has many time varying parameters, but some of the most important for the quality of the material processing are given by the directing part of the equipment which contains the focusing and deflecting systems. These systems modify the beam diameter, focus distance and beam penetrations in the material.

One of the first stages for the control system design is the modelling and simulation of the process, in this case the focusing and deflecting systems. The study and simulations results help the engineers to know the process characteristics and what type of control is needed. After simplifying, the resulted focusing model is nonlinear and the deflecting model is considered linear on small angles. The simulations shown in the paper are obtained using Matlab environment.

1. Introduction to electron beam equipment and control

The CTW 5/60 is an electron beam equipment (EBP) used in high quality material processing as welding, cutting and thermal surface treatment process. This type of non-conventional technique solves great topical problems in industrial manufacturing, wherever conventional techniques proved to be inefficient. In fact, electron beam and laser are the only ways of delivering large amounts of concentrated thermal energy to materials (maximum 10^8 W/cm^2).

Electron beam material processing involves many complex phenomenons like the electrons generation, beam forming and transport, heat absorption in the workpiece, but the special electron beams properties like high resolution, long depth of field attainable, high power energy density make it very useful in material handling. Nuclear technologies, aeronautics, microelectronics are some examples where this equipment is used.

The desired performances are obtained with a precise heating effect of the material, which depends on the position of the electron beam spot. The directing parts of the EBP equipment used in this case (for constant values of the main processing parameters: power, accelerating voltage and working distance) are the magnetic focusing and deflecting components.

Figure 1 shows the electron beam equipment without digital interfaces, drivers and central unit.

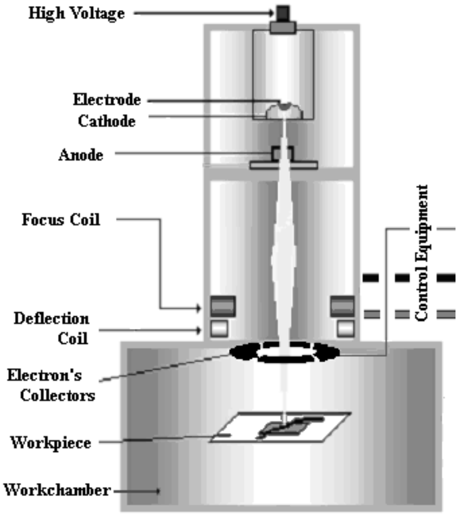

Figure 1. Electron beam equipment

The main parts of the equipment are the triode gun and the vacuum system that provides high vacuum environment.

The triode gun design consists of the cathode, composed of the filament and the massive cathode, electrode or grid, anode, focusing and deflecting coils. The vacuum system ensures a pressure level of 10^{-3}–10^{-4} Pa and it is controlled by a multitasking digital system implemented on the microcontroller and on PC. To avoid accidents, any error that may appear in this unit is pointed out and preparing sequences for material processing are halted.

Keywords: *electron beam, deflecting system, simulation*

The emission of electrons from the incandescently heated termoemission filament, which is saturated during the process by a predetermined amount of electrical current, generates the main beam. A negative high voltage potential is applied to the filament cathode assembly, referred to as the accelerating voltage of 40…60 kV. A lower voltage than the accelerating voltage is applied to the grid cup. In this way the grid cup acts as a valve that controls the volume of electron energy, which can flow from the cathode to attracting targets.

The first target, situated in the triode gun, is an anode at a positive potential, which forms the beam. Then the focused beam of electrons is led using focusing coil to a secondary target, situated in the workchamber, consisting of a metallic workpiece, where the kinetic energy of the electrons is converted into thermal energy.

The metallic workpiece offers a conductive path to earth to complete the circuit. This target can be stationary and the electron beam energy deflected using deflecting coil or the workpiece can be moved using a CNC table.

The magnetic focusing coil is located beneath the anode assembly and is circular in design and concentric with electron beam. An electrical current is passed through the coil, which produces magnetic fluxes that provides the convergence of electron beam. The deflecting coil is created with four wound coils positioned at right angles to the column.

Another important part in the experimental equipment is the electron's collectors composed of four electrodes used to capture electrons reflected from the target surface (workpiece).

The paper presents the study of the influence of these two directing components on the electron beam position and some models and simulations for the CTW 5/60 equipment.

So, these final stages of the electron beam equipment are very important because the trajectory on the surface of a stationary workpiece is obtained using the deflecting systems and the desired depth of penetration (depth/width ratio) is achieved with the aid of the focusing system.

Electron Beam CTW 5/60 Equipment is developed by "Petru Maior" University of Tîrgu Mureş in partnership with Electrical Research Institute I.C.P.E. Bucharest.

2. Focusing and Deflecting Systems. Mathematical modelling

2.1. Electrons movement equations

In the electron beam gun the electrons trajectory is influenced by electric and magnetic fields. We suppose that these fields satisfy the Maxwell requirements. So, using the Newton's second law of motion results the general equation of the electrons motion in electric and magnetic fields:

$$\frac{d^2\overline{r}}{dt^2} = \frac{e_0}{m_e}\left(\overline{E} + \overline{v}_e \times \overline{B}\right) \tag{1}$$

where m_e is the electron mass, $9.1\cdot10^{-31}$ [kg], r is the electron position in the space and e_0/m_e characterizes the electrons mobility, $1.76\cdot10^{11}$ [C/kg].

Configuration of the electron beam equipment presents two perpendicular fields, focusing magnetic field across the electric field. At the electron beam equipment the components of the magnetic field (B_x, B_y), transversal to the electrono-optical axis, are the products of the two deflecting coils to deviate the EB from this axis, while the electric field (component E_z) acting on the electrons has the purpose to accelerate it using the potential difference between cathode and anode, respectively the workpiece potential.

So, the electrons motion equations in rectangular coordinates are:

$$\begin{cases} \dfrac{d^2x}{dt^2} = \dfrac{e_0}{m_e}\cdot\dfrac{dz}{dt}\cdot B_y \\ \dfrac{d^2y}{dt^2} = \dfrac{e_0}{m_e}\cdot\dfrac{dz}{dt}\cdot B_x \\ \dfrac{d^2z}{dt^2} = \dfrac{e_0}{m_e}\cdot\left(E_z + \dfrac{dx}{dt}\cdot B_y - \dfrac{dy}{dt}\cdot B_x\right) \end{cases} \tag{2}$$

The magnetic fields can be produced also by the focusing coils to concentrate the energy of the electrons in the desired focal spot.

2.2. Focusing system mathematical modelling

The focusing system is one of the most important control systems of the electron beam equipment automation.

Including the magnetic field distribution and using some optics concepts from the dynamic equations of the electrons that cross through the electromagnetic coil results the stationary model of the focusing system, which is a relation between the focusing distance and focusing current.

The electrons have a screw motion and so the equations (2) were transposed in cylindrical coordinates:

$$\begin{cases} \dfrac{d^2r}{dt^2} - r\cdot\left(\dfrac{d\theta}{dt}\right)^2 = \dfrac{e_0}{m_e}\cdot\left(-\dfrac{\partial V}{\partial r} + r\cdot\dfrac{d\theta}{dt}\cdot B_z - \dfrac{dz}{dt}\cdot B_\theta\right) \\ r\cdot\dfrac{d^2\theta}{dt^2} + 2\cdot\dfrac{dr}{dt}\cdot\dfrac{d\theta}{dt} = \dfrac{e_0}{m_e}\cdot\left(-\dfrac{1}{r}\cdot\dfrac{\partial V}{\partial\theta} + \dfrac{dz}{dt}\cdot B_r - \dfrac{dr}{dt}\cdot B_z\right) \\ \dfrac{d^2z}{dt^2} = \dfrac{e_0}{m_e}\cdot\left(-\dfrac{\partial V}{\partial r} + \dfrac{dr}{dt}\cdot B_\theta - r\cdot\dfrac{d\theta}{dt}\cdot B_r\right) \end{cases} \tag{3}$$

where z, r, θ are the cylindrical coordinates of the electrons position.

Because the focusing coil act like a thin lens and considering a limited effect of the coil the equations are:

$$\begin{cases} \dfrac{d^2r}{dt^2} = r\cdot\left(\dfrac{d\theta}{dt}\right)^2 + \dfrac{e_0}{m_e}\cdot\left(r\cdot\dfrac{d\theta}{dt}\cdot B_z - \dfrac{dz}{dt}\cdot B_\theta\right) \\ \dfrac{d^2\theta}{dt^2} = 0 \\ \dfrac{d^2z}{dt^2} = \dfrac{e_0}{m_e}\cdot\left(\dfrac{dr}{dt}\cdot B_\theta - r\cdot\dfrac{d\theta}{dt}\cdot B_r\right) \end{cases} \tag{4}$$

If the axial component B_z of the magnetic field is approximated with the Taylor series and if the angular component B_θ of the field is null for concentric coils, from the Laplace equation results the radial component B_r.

Using these components the analytical equations of the electrons trajectory are given in the relation (5).

$$\begin{cases} \dfrac{d\theta}{dt} = -\dfrac{e_0}{m_e}\cdot\dfrac{B_0(z)}{2} \\ \dfrac{d^2r}{dt^2} = -\left(\dfrac{e_0}{m_e}\right)^2\cdot r\cdot\dfrac{B_0^2(z)}{4} \\ \dfrac{d^2z}{dt^2} = -\left(\dfrac{e_0}{m_e}\right)^2\cdot\dfrac{r^2}{4}\cdot B_0(z)\cdot\dfrac{dB_0(z)}{dt} \end{cases} \tag{5}$$

For a very small value of the square of the radius r the variation d^2z/dt^2 is zero. On the other hand the electrons which reach the lens have already acquired kinetic energy and the Oz velocity $v_z=dz/dt$ is dependent on the voltage acceleration U_{acc}. The axial motion of the electrons became:

$$\frac{d^2r}{dz^2} = -\frac{e_0}{m_e}\cdot\frac{r\cdot B_0^2}{8\cdot U_{acc}} \tag{6}$$

where U_{acc} is the accelerating voltage and B_0 is the magnetic field distribution.

Integrating the relation (6) around the thin lens, considering a parallel beam to Oz axis and knowing the magnetic induction distribution created by the circular coil the focusing distance is:

$$z_{foc}(i_{foc}) = \frac{256}{3}\cdot\frac{m_e}{\pi\cdot e_0\cdot\mu_0^2}\cdot\frac{R\cdot U_{acc}}{(n\cdot i_{foc})^2} \tag{7}$$

where z_{foc} is the focusing distance, R the radius of the circular focusing coil, n the number of turns, i_{foc} the focusing current and μ_0 the permittivity.

The figure 2 shows the analogy between the focusing magnetic coils and thin lenses, respectively the optics concept.

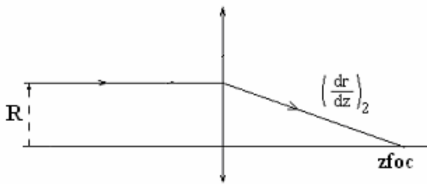

Figure 2. The beam crossing the thin magnetic lens

Relation (7) consists in the nonlinear dependencies between the number of turns n, focusing current i_{foc} and focusing distance z_{foc}.

The dynamical model of the focusing system, actually the second part of the focusing mathematical model, is the dependency between the prescribed focusing coil voltage and the focusing current.

$$L_s\frac{di_{foc}}{dt} + R_s i_{foc}(t) = u_{foc}(t) \tag{8}$$

where L_s and R_s are the characteristics parameters of the focusing coil.

The velocity of electrons is very high and for this reason the time that has been elapsed from the modification of the coil magnetic field and changes in focus distance is too small (the order of nanoseconds) and can be neglected. Dynamic focusing system is thus determined only by the focusing coil current changes.

2.3. Deflecting system mathematical modelling

Deflecting system provides spatial deviation of the electron beam so that it is possible to reach any point on the surface of the target material. Whatever type of deflecting system, it must be characterized by sensitivity, linearity and small influence on the focusing system. Aberrations due deflection became significant for deflecting angles greater than 10-15°.

The deflecting system has a mathematical model with a stationary part and a dynamic part. The stationary model is a particular solution of the dynamic equations of the electrons when the electric and magnetic field distribution is known and gives the dependence between the one dimension deflecting distance and the deflecting coil current.

For one axis deflection the magnetic field is transversal ($v\perp B$), Lorentz force is actually a centripetal force that respects the relation (9) and the electrons describe a circular motion with radius R shown in figure 3.

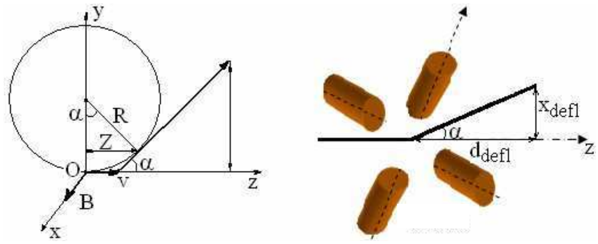

Figure 3. The beam crossing the deflecting coils

$$\frac{m_e \cdot v_z^2}{R} = e_0 \cdot v_z \cdot B \tag{9}$$

The deflecting angle of the electrons in the transverse magnetic field can be determined using the geometric relation (10).

$$\sin(\alpha) = \frac{Z}{R} = \frac{e_0 \cdot B \cdot Z}{m_e \cdot v_z} \tag{10}$$

where R is the radius, v_z the electron velocity into the magnetic field, Z the length of the region where the magnetic field acts uniformly (equal to the length of deflecting coil l_b) and B the magnetic induction.

The velocity of electrons depends on the accelerating voltage:

$$v_z = \sqrt{\frac{2 \cdot e_0 \cdot U_{acc}}{m_e}} \tag{11}$$

For small deflecting angles the sinus function can be replaced with the tangent function and using another geometric relation results the deflection (12).

$$x_{defl} = d_{defl} \tan(\alpha) \approx \sqrt{\frac{e_0}{2 \cdot m_e}} \cdot \frac{B \cdot d_{defl} \cdot l_b}{\sqrt{U_{acc}}} \tag{12}$$

The magnetic induction B is given by relation (13).

$$\mathbf{B}(i_{xdefl}) = \frac{k_b \cdot \mu_0 \cdot n \cdot i_{xdefl}}{a} \tag{13}$$

where k_b is the constant approximation of the induction curve, n the number of turns, a the constant dispersion of the magnetic field as the axis of symmetry, i_{xdefl} the deflecting coil current.

Inserting the relation (13) in the relation (12) results the stationary deflecting mathematical model (14).

$$x_{defl} \approx \sqrt{\frac{e_0}{2 \cdot m_e}} \cdot \frac{k_b \cdot \mu_0 \cdot d_{defl} \cdot l_b}{a \cdot \sqrt{U_{acc}}} \cdot n \cdot i_{xdefl} \tag{14}$$

The stationary deflecting mathematical model for the other axis is similar and depends on the other deflecting coil current.

The dynamical model of the deflecting system is similar to the dynamical model of the focusing system and contains a first order differential equation between the deflecting current and the voltage supplied to the deflecting coil.

$$L_s \frac{di_{xdefl}}{dt} + R_s i_{xdefl}(t) = u_{defl}(t) \tag{15}$$

where L_s and R_s are the characteristics parameters of the deflecting coil.

2.4. Simulink models for the focusing and deflecting systems

Relations (7), (8), (14) and (15) give the electron beam directing mathematical models. These models have been implemented in Matlab Simulink for later simulations.

Figure 4 shows the Simulink model of the electron beam focusing system.

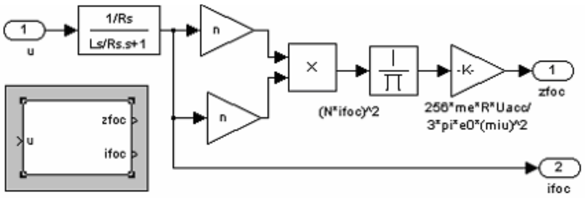

Figure 4. Focusing system simulink model

Figure 5 shows the simulink model of the electron beam deflecting system.

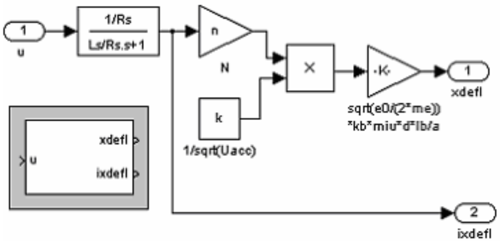

Figure 5. Deflecting system simulink model

Both simulink models (from figures 4 and 5) contain the stationary and the dynamic parts. The dynamic part was converted in a first order transfer function using basic Laplace transforms and implemented with an appropriate simulink block.

Note also that while de focusing stationary model is nonlinear, the deflecting stationary model is linear. This observation may have important consequences on the adopted control type. Also, the electron beam deflection on small distances is recommended to eliminate the mechanical inertia introduced by the d.c. motors.

3. Focusing and deflecting systems simulations

The screw motion of the electrons in the electron beam gun was first simulated using the relation (5) and figure 5 shows such an example in the three dimensional space.

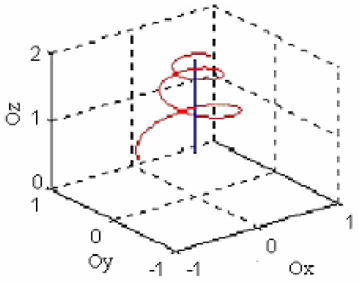

Figure 6. Electron beam trajectory

Replacing the inductance $L_s=0.450H$ and the resistance $R_s=16\Omega$ in the relation (8), dynamic focusing model was tested at a 1V step input. The focusing current response (Figure 7) has a first order system response with a 0.0625 amplification factor and a 0.028 seconds time constant.

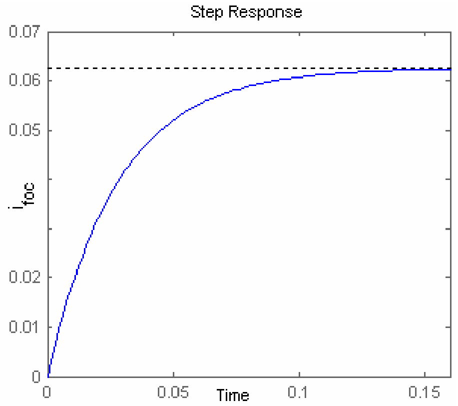

Figure 7. Focusing current response at step focusing voltage input

Relation (7) gives the focusing distance z_{foc} depending on the focusing current i_{foc} and beam focusing coil radius R. So, we calculated the focusing distance for three different values of coil radius. The variation of the focusing current was kept in the practical range 0.2...0.8A.

The figure 8 emphasizes again the nonlinear dependency of the stationary focusing model. In the simulations we used for the other parameters the following values: $n=1024$ *turns*, $m_e=9.1\cdot10^{-31}kg$, $U_{acc}=60kV$, $\mu_0=4\pi10^{-7}Vs/Am$ and $e_0=1{,}6\cdot10^{-19}C$.

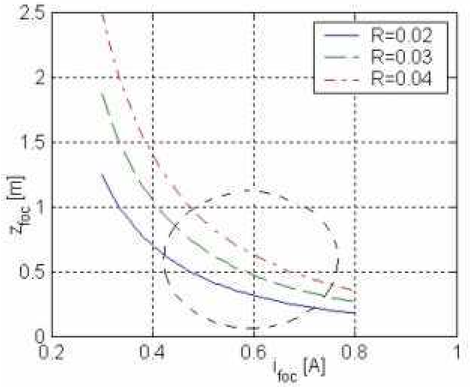

Figure 8. Focusing distance-focusing current nonlinear dependency

The measured inductance and resistance for the dynamic deflecting model are $L_s=0.450H$ and $R_s=10\Omega$. The deflecting current response at a 1V step input is a first order system response, too. The figure 9 indicates the 0.1 amplification factor and the 0.045 seconds time constant.

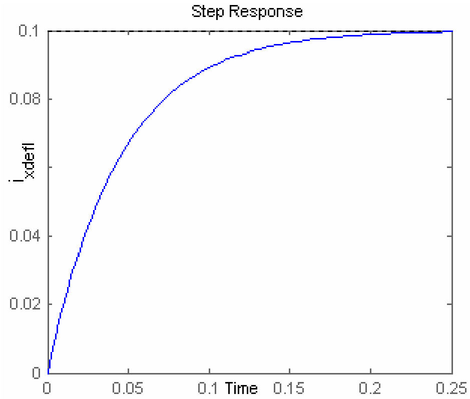

Figure 9. Deflecting current response at step deflecting voltage input

Relation (14) gives the deflecting distance x_{defl} depending on the deflecting current i_{xdefl} and distance from the deflecting coil to the target d_{defl}. We calculated the deflecting distance for three different values of the distance d_{defl}. The variation of the deflecting current was kept under the 0.25A value.

The figure 10 highlights the linear dependency of the stationary deflecting model. In the simulations we used for the other parameters of the stationary deflecting model the following values: $k_b=0.4$, $a=0.06$, $l_b=0.06m$, $n=1000$ *turns*.

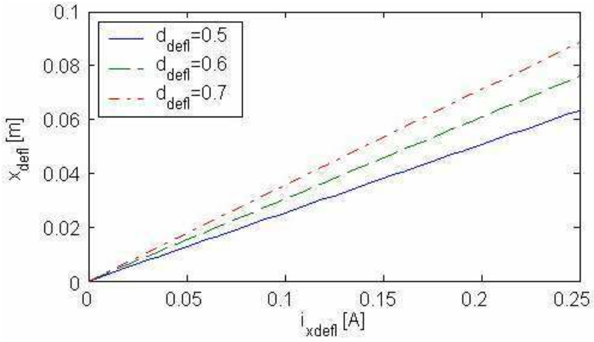

Figure 10. Deflecting distance-deflecting current linear dependency

4. Conclusions

The paper presented the modelling and simulations of the electron beam directing systems which contains the focusing and deflecting components.

In the electron beam equipment the focusing system ensures the concentration of the electron beam energy into a focal point in depth of the material, while the deflecting system ensures a desired trajectory to the target material surface.

Knowing the magnetic field distribution and using some optics concepts from the dynamic equations of the electrons that cross through the electromagnetic coil we proposed the stationary model of the focusing system. The stationary model of the focusing system (relation (7)) indicates the nonlinear dependency between the focusing current and focusing distance.

The resulted stationary model of the deflecting system is a particular solution of the dynamic equations of the electrons which traverse the deflecting coil. The dependence between the deflecting coil current and the one dimension deflecting distance is approximated with the linear relation (14).

An important advantage of placing the magnetic deflection is to eliminate mechanical inertia. At the same time we observed by comparing the two stationary models (nonlinear focusing and linear deflection) the need to introduce in the future different type of control systems.

The dynamic models of the focusing and deflecting systems, relations (8) and (15), are similar and are represented by the first order elements (first order differential equations) that differ only by the parameters. Both models consider the voltage as input and current as output.

These electron beam directing models, with the measured parameters and data from the technical catalog, was implemented and simulated in the Matlab Simulink environment and the results are close to the real values.

The mathematical models and the Simulink blocks can be used in future works to improve the directing control systems, in particular and the quality of the electron beam processing, in general.

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