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STUDYING AND MODELING VIBRATION TRANSDUCERS AND ACCELEROMETERS

Katalin Ágoston
”Petru Maior”University of Târgu Mureş, Romania
kagoston@engineering.upm.ro

ABSTRACT

This paper presents types and operating mode of vibration sensors. Piezoelectric sensing elements are often used in accelerometers. It will be investigate the structure and transfer function of the seismic mass type sensing element. The article presents how the piezoelectric sensing element works and how can be modeled with an electronic circuit. The transfer functions of the electronic circuit models are studied in Matlab and the results are presented. It will be presented the influence of the seismic mass on the accelerometer's working frequency domain.

Keywords: piezoelectric sensors, vibration transducers, Matlab, Simulink model

1. Introduction

Vibration and shock are present in all areas of our lives. They may be generated and transmitted by motors, turbines, machine-tools, bridges, towers, and even by the human body. Some vibrations are desirable; others may be disturbing or even destructive. Consequently, there is often a need to understand the causes of vibrations and to develop methods to measure and prevent them.

Accelerometers can be used in a wide variety of low g applications such as tilt and orientation, vibration analysis, motion detection, etc. There are many types of sensors that measure acceleration, vibration, shock, or tilt. These sensors include piezo-film, electromechanical servo, piezoelectric, liquid tilt, bulk micromachined piezo-resistive and capacitive sensors, as well as surface micromachined capacitive sensors. Each of these sensors has distinct characteristics in the output signal of the sensor. Measurement of acceleration can also provide velocity by single integration and position by double integration. Vibration and shock can be used for machine health determination as well as motion and shock detection for car alarms, airbags, and sports training products. Static acceleration due to gravity can be used to determine tilt and inclination.

The piezoelectric accelerometers are widely accepted as the best choice for measuring absolute vibration. Compared to the other types of sensors, piezoelectric accelerometers have important advantages: extremely wide dynamic range, almost free of noise - suitable for shock measurement as well as for almost imperceptible vibration, linearity over their dynamic range, wide frequency range, compact and highly sensitive, no moving parts - long service life, self-generating - no external power required, great variety of models available for nearly any purpose, integration of the output signal provides velocity and displacement determination. The disadvantage of piezoelectric sensors, only alternating acceleration can be measured. This type of accelerometer is not capable of true dc response, e.g. gravitation acceleration. The sensor has a high impedance output what needs to be converted first into a low impedance signal. For sensors with charge output, an external charge amplifier is required. For processing the sensor signal, a variety of equipment can be used, such as: RMS and peak value meters, frequency analyzers, recorders, PC instrumentation. However, the capability of such equipment would be wasted without an accurate sensor signal. In many cases the accelerometer is the most critical link in the measurement chain. To obtain precise vibration signals some basic knowledge about piezoelectric accelerometers is required. [1], [6]

2. The vibration sensor

The displacement, the velocity and the acceleration are bond. If we know the equation which describes the motion of a piece (of a body) due to a force, we can provide through single derivation the velocity and acceleration by double derivation. In practice we measure the acceleration with a proper sensor and we get velocity and displacement through integration, because integration circuits are most simple.

The acceleration transducers have two parts: sensing element and conversion circuit. The sensing element is a seismic mass type and it's fixed on the moving body and takes over the movement precisely (Fig.1). The movement of the seismic mass can be described with the equation:

$$m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky=F\cdot a=-m\frac{d^2x}{dt^2} \qquad (1)$$

Where m is the seismic mass, k is springiness coefficient, c is the damping coefficient, a is the acceleration and F is the force. [8]

The transfer function of the sensor's system is:

$$H(s)=\frac{Y(s)}{X(s)}=\frac{-s^2}{s^2+2\xi\omega_0 s+\omega_0^2} \qquad (2)$$

Where $\omega_0=\sqrt{k/m}$ has the meaning of resonance pulse and $\xi=\frac{c}{2\sqrt{km}}$ damping factor.

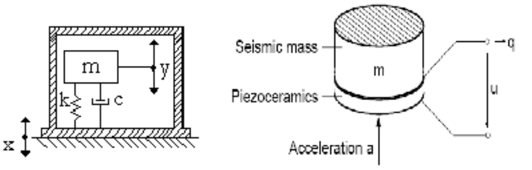

Fig.1. *Sensing element's general structure and piezoelectric sensing element.*

Depending on the relation between m, k and c the movement of the seismic mass is proportional with displacement, or velocity, or acceleration of the body. For acceleration measurement m and c are small, negligible against k.

$$ky\approx -m\frac{d^2x}{dt^2}\Rightarrow y\approx -\frac{m}{k}\frac{d^2x}{dt^2} \qquad (3)$$

The sensing element of a piezoelectric accelerometer, the piezoceramic material, is sandwiched between two electrodes. A force applied perpendicular to the disk causes a charge production and a voltage at the electrodes. Since the seismic mass is constant the charge output signal is proportional to the acceleration of the mass. [2] [6]

The charge sensitivity of the piezoceramic material is $S_q=q/a$, and the voltage sensitivity $S_q=u/a$.

Over a wide frequency range both sensor base and seismic mass have the same acceleration magnitude. Hence the sensor measures the acceleration of the test object.

A piezoelectric accelerometer can be regarded as a mechanical low-pass filter with resonance peak. The seismic mass and the piezoceramics form a spring mass system. The resonance frequency of the system defines the upper frequency limit of an accelerometer. To achieve a wider operating frequency range the resonance frequency should be increased. This is usually done by reducing the seismic mass. However, the lower the seismic mass, the lower the sensitivity. Figure 2 shows a typical frequency response curve of an accelerometer when it is excited by a constant acceleration. [5], [7]

At approximately 1/5 of the resonance frequency the response of the sensor is 1.05. This means that the measured error compared to the lower frequency is 5%.

At approximately 1/3 of the resonance frequency the error is 10%. For this reason the "linear" frequency range should be considered limited to 1/3 of the resonance frequency. The 3dB limit with approximately 30% error is obtained at approximately one half times of the resonance frequency.

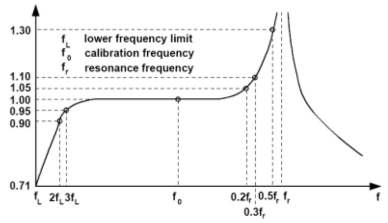

Fig.2. *Frequency response of an accelerometer.*

The output of the sensor can be a charge or a voltage. Accelerometers with charge output generate an output signal in the range of some pC with very high impedance. To process this signal by standard ac measuring equipment it needs to be transformed into a low impedance voltage signal with a charge amplifier. The input stage of a charge amplifier features a capacitive feedback circuit which balances the effect of the applied charge input signal. The feedback signal is then a measure of input charge (Fig.3).

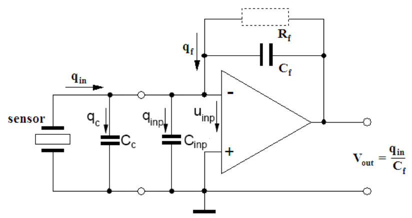

Fig.3. *Charge amplifier.*

The C_c is the cable capacitance, C_{inp} is the amplifier input capacitance and C_f the feedback capacitor. We can assume that the input voltage of the charge amplifier u_{inp} is equal to zero and we get an output voltage proportional to the input charge. The feedback resistor R_f has the function to provide dc stability to the circuit and to define the lower frequency limit of the amplifier (the RC time constant). The voltage signal can be transmitted over longer distances.

The charge amplifier and other added circuits can form a built-in circuit powered by a constant current source (Fig.4). The vibration signal is transmitted back to the supply as a modulated bias voltage. Both supply current and voltage output are transmitted via the same coaxial cable (IEPE transducer). The capacitor C_C is a coupling capacitor and removes the sensor bias voltage from the instrument input providing a zero-based ac signal. Cc and R_{inp} acts as a high pass filter, its time constant should be sufficiently high to let all relevant low frequency components of the sensor signal pass. The output impedance is typically 100 - 300Ω. The dc output voltage of the sensor without excitation is between 8 and 12 V and it varies with supply current and temperature. In most applications voltage output accelerometers are used. [3], [7]

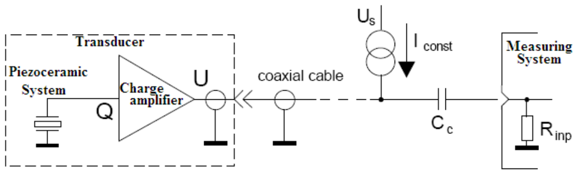

Fig.4. *IEPE accelerometers.*

Charge sensitivity or voltage sensitivity describe the relationship between acceleration and output. The voltage sensitivity is expressed in mV/g or mV/ ms^{-2}. [4] [8]

3. Modeling the piezoelectric sensor

The piezoelectric sensor has very high output impedance and can be modeled as a voltage source in series with the sensor's capacitance (C_S), or a charge source in parallel with the capacitance (Fig.5). The voltage *V* at the source is directly proportional to the applied force, pressure, or strain.

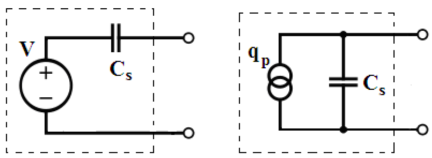

Fig.5. *Piezoelectric sensor models.*

The detailed model of the sensor includes the effects of the sensor's mechanical construction and other elements. The inductance L_m represents the seismic mass and inertia of the sensor, C_e is inversely proportional to the mechanical elasticity of the sensor, C_0 represents the static capacitance of the transducer, R_i is the insulation leakage resistance of the sensor element. If the sensor is connected to a load resistance (input resistance of the measurement circuit), this also acts in parallel with the insulation resistance, both increasing the high-pass cutoff frequency of the sensor (Fig.6a).

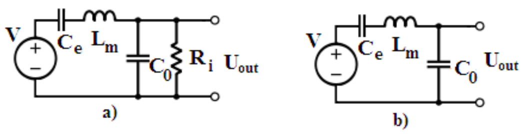

Fig.6. *Schematic symbol and electronic model.*

To investigate the behavior of the sensor we use the transfer function of the electronic model.

$$H(s)=\frac{Y(s)}{X(s)}=\frac{\frac{R_i\cdot C_e}{L_m\cdot C_e+1}s}{\left(\frac{R_i\cdot C_e}{L_m\cdot C_e+1}+C_0\right)s+1}=\frac{a\cdot s}{(a+C_0)s+1} \quad (4)$$

The frequency response of the system and the influence of the R_i resistance, can be observed on the Bode diagram (Fig.7).

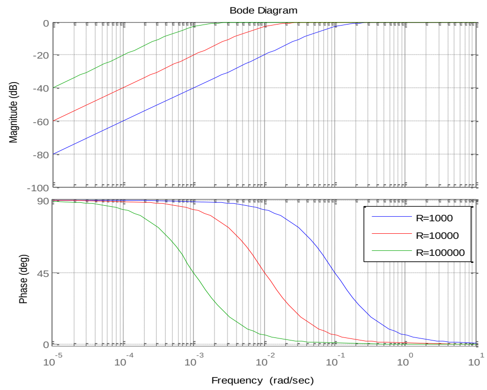

Fig.7. *The frequency response at different R values.*

Analyzing the time response of the piezoelectric sensor model circuit we can observed the output fallows perfectly the input (Fig.8); this means the output voltage is the measure of the input signal.

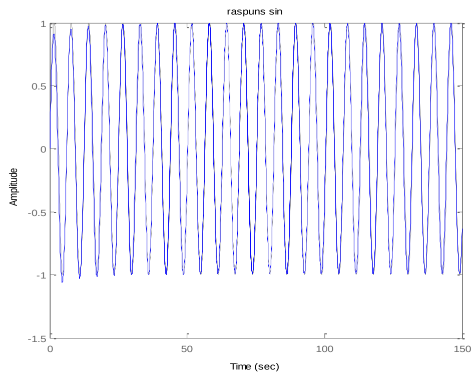

Fig.8. *System response to a sin excitation.*

Studying the accelerometer electronic model without the R_i resistance (Fig.6b) in frequency domain we can observe the same behavior as shown previously. The transfer function of this model is:

$$H(s)=\frac{Y(s)}{X(s)}=\frac{s}{L_mC_0s^3+\left(\frac{C_0}{C_e}+1\right)s}=\frac{s}{a_1s^3+b_1s}$$

(5)

Where L_mC_0 represent the resonant frequency of the system (ω_0) and influences the frequency domain of the accelerometer.

For $\omega<\omega_0$ the sensor can be used as an accelerometer, in this frequency range the damping of the input signal is zero. The variation of the input signal is taking over precisely. For $\omega\cong\omega_0$ the sensor is functioning as frequency meter and for $\omega>\omega_0$ the sensor is functioning as a vibrometer. If the seismic mass (L_m) of the sensor decreases the frequency domain of the accelerometer rises (Fig.9). Generally the accelerometer is calibrated for $\omega=0.3\omega_0$.

The accelerometer used at higher frequency has a smaller seismic mass, but the sensitivity is likewise smaller. For example an accelerometer with m=10-50g has a charge sensitivity 1-10pC/ms^{-2} and an accelerometer with m=0.5-3g has a charge sensitivity 0.1-0.3pC/ms^{-2}.

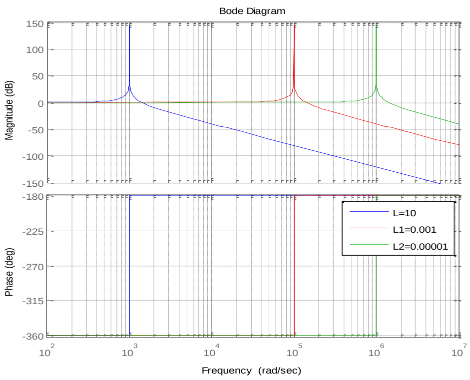

Fig.9. *The frequency response of the electronic model without R_i at different L_m values.*

The sensor considered as a system can be studied and modeled through the equation which described the electronic circuit model. The equation suitable for the circuit from figure 6b is:

$$u_{out} = u_{C0} = \frac{1}{C_0}\int i dt \qquad (6)$$

$$V = u_{Ce} + u_{Lm} + u_{C0} = \frac{1}{C_e}\int i dt + L_m \frac{di}{dt} + \frac{1}{C_0}\int i dt$$

The proper Simulink model is presented on figure 10. The output signal is the voltage on the C_0 capacitor.

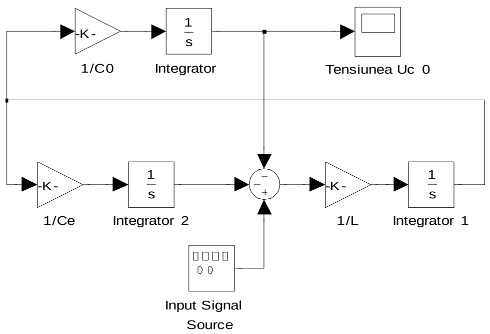

Fig.10. *Simulink model for the piezoelectric sensing element.*

The piezoelectric sensing element is connected usually at the input of a conditioning circuit. Considering the input resistance of this circuit and the output resistance of the sensing element, the electronic circuit model becomes as shown in figure 6a. The equation suitable for the circuit is:

$$u_{out} = u_{Ri} = R_i \cdot i_R = R(i - i_{C0}) \qquad (7)$$

$$i_{C0} = C_0 \frac{du_{C0}}{dt}$$

$$u_{C0} = V - u_{Ce} - u_{Lm} = V - \frac{1}{C_e}\int i dt - L_m \frac{di}{dt}$$

The proper Simulink model is presented on figure 11. The output signal is the voltage on the R_i resistance.

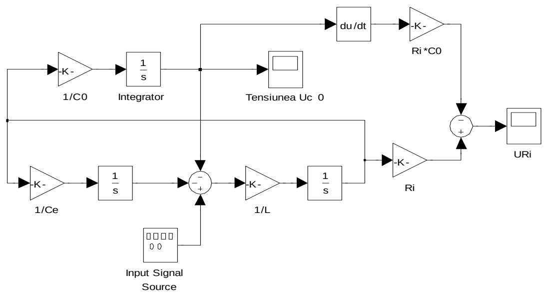

Fig.11. *Simulink model with R_i resistance.*

The both models give the same results.

4. Conclusions

The piezoelectric accelerometers are widely used for measuring absolute vibration. Compared to the other types of sensors, piezoelectric accelerometers have important advantages, but to obtain precise output vibration signals some basic knowledge about piezoelectric accelerometers is required. In many cases the accelerometer is the most critical part in the measurement equipment.

It is presented the general structure of the sensing element and the equivalent electronic circuit model. Simulink models are developed with the proper equations. Transfer functions are used to simulate the frequency response of the electronic models with and without R_i at different L_m (seismic mass) values.

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