

## FEM ANALYSIS OF STRESS AND STRAIN STATE OF A WORM FACE-GEAR WITH REVERSE TAPERED PINION

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### ABSTRACT

*In this paper an analysis of stresses and strains of the condition of a worm face-gear with reverse tapered pinion. The study is performed by FEM analysis.*

**Keywords:** Worm-Face Gear, Reverse tapered, FEM analysis, Finite Element, Stresses, Displacements

### 1. Theoretical aspects

The worm face-gear (Fig.1.) belonging to the group of gear with cross axis is defined as a worm face-gear with reverse taper pinion.

Compared to the other gears these face-gear presented many constructive and functional advantages: increased bearing force to general-purpose worm gear which makes them to be sensitive overall dimensions smaller, contact line incomparably high coverage, good lubrication conditions, higher ratios, increased accuracy and quiet operation.

Due to the particular geometry of worm face-gear with reverse taper pinion - the flanks of worm being strongly asymmetric approach, allows the absolute control of the backlash, the axial displacement of a worm or screw worm-wheel.

The characteristic geometry elements of a worm face-gear with reverse tapered are presented in Fig 2.



Fig. 1. The worm face-gear with reverse tapered pinion

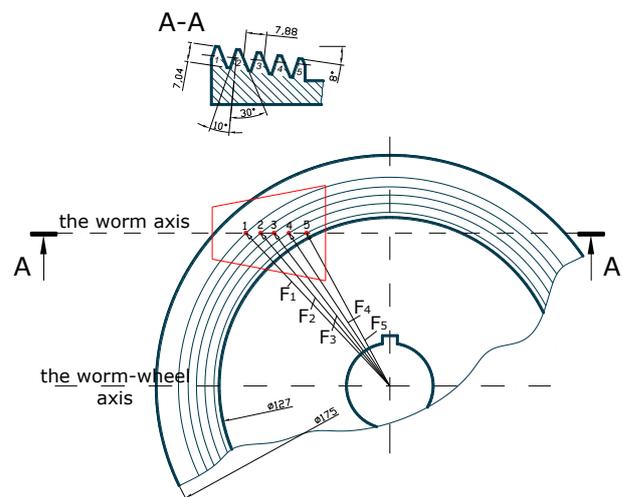


Fig. 2. The constructive geometry of worm face-gear. The contact position of worm-wheel in operation

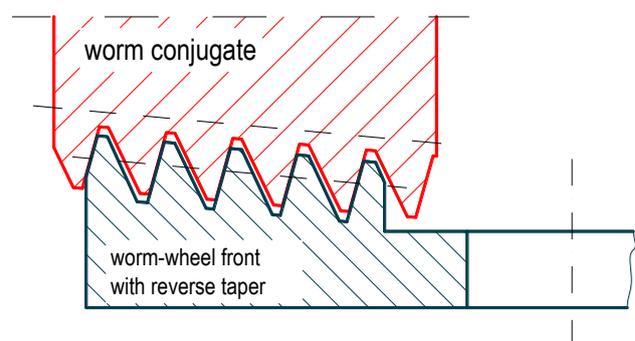


Fig.3. Section through the worm worm-wheel; the bearing flank 10°

## 2. Modeling and meshing

Having analyzed the studied structure, it was found that a correct interpretation of the phenomenon of contact between worm-worm wheels can be seen in axial section plan between the two elements of face-gear (Fig. 2).

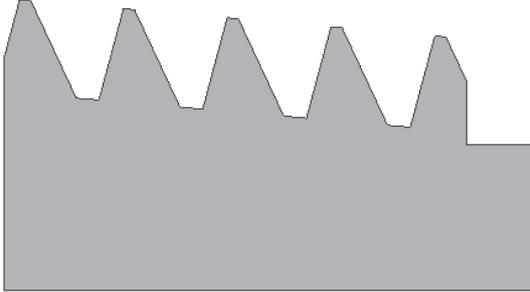


Fig. 4. Modeling – Inventor 10

Geometric data corresponding section of the plan was made in program design modeling Inventor.10 (Fig. 4) and then made the transfer to ALGOR V16 Fempro analysis program (Fig.5).

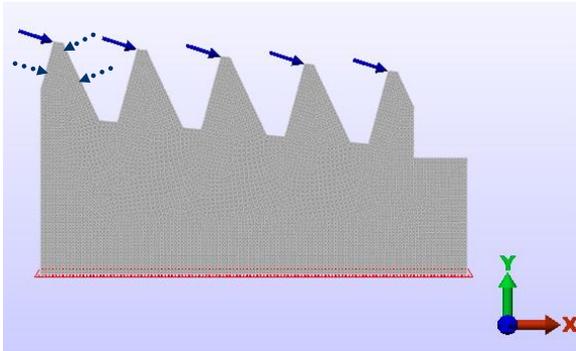


Fig.5. Meshing – Loads - Constraints

Finite elements used are type 2D - two-dimensional quadrilateral and triangular. After meshing result:

- a. Number of nodes : 12,989
- b. Number E.F.- type 2D: 13,042

Nodal boundary conditions are fixed in the nodes of the worm-wheel.

The features elastic material of wheel which is by: ERTACETAL C, the elastic characteristics:  $E = 3600 \text{ N/mm}^2$ ,  $\mu = 0.4$  [7].

The loading flank's was done by calculating the forces corresponding to the 5 points of contact between teeth of worm-wheel in contact with the conjugated surface of the worm (Fig.3).

The torque transmitted of the worm-wheel  $M_t$  was calculating with relationship:

$$M_t = 9,55 \cdot 10^6 \frac{P}{n} \text{ [N.mm]} \quad (1)$$

Which:

- $P$  - power [kW];
- $n$  - speed [r.p.m];

On the other hand the torque transmitted  $M_t$  is:

$$M_t = F \cdot \frac{d}{2} \quad (2)$$

Which:

- $F$  - force in contact point of worm-worm-wheel [N];
- $d$  - diameter of contact point corresponding for each tooth of the worm wheel;

$$\Rightarrow F = \frac{2}{d} \cdot 9,55 \cdot 10^6 \frac{P}{n} \text{ [N]} \quad (3)$$

Considered the following corresponding to experimental values:

$P = 1,5 \text{ kW}$  and  $n = 1500 \text{ rpm}$  input speed, gear ratio  $i = 1:47$ , axial distance  $A = 58 \text{ mm}$ , module  $m_a = 2,5 \text{ mm}$ , bearing flank angle  $\alpha_1 = 10^\circ$ , support flank angle  $\alpha_2 = 30^\circ$ , Archimedes worm type reverse tapered,  $\delta_1 = 5^\circ$ , reverse tapered worm-wheel drive  $\delta_2 = 8^\circ$ .

Thus, corresponding the diameter of contact points of five teeth of worm-wheel results:

- 1 -  $d_1 = 158,35 \text{ mm} \Rightarrow F_1 = 120,62 \text{ N}$
- 2 -  $d_2 = 151,06 \text{ mm} \Rightarrow F_2 = 126,44 \text{ N}$
- 3 -  $d_3 = 144,84 \text{ mm} \Rightarrow F_3 = 131,87 \text{ N}$
- 4 -  $d_4 = 138,46 \text{ mm} \Rightarrow F_4 = 137,95 \text{ N}$
- 5 -  $d_5 = 131,70 \text{ mm} \Rightarrow F_5 = 145,03 \text{ N}$

Constructive and functional reasons, the tooth five is considered a hard drive smaller force than that calculated, namely:  $F_5 = 130,5 \text{ N}$ .

## 3. Results

The results are considered to be significant for the interpretation and relevance given the phenomenon studied, are the values:

- Stress by Von Mises equivalent theory  $\sigma_{ech}$ ;
- Minimal by main stress  $\sigma_2$  (with compression effect);
- Resultant structure shift. Displacement.

They studied four cases of loading:

Case I - load bearing flank to  $10^\circ$ :

- a) Entry into gear
- b) Loading in operating

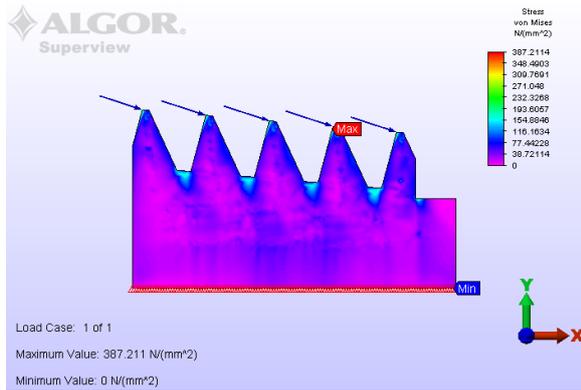
Case II - load bearing flank to  $30^\circ$ :

- a) Entry into gear
- b) Loading in operating

On the simulations performed were obtained the following results:

**Case I - load bearing flank to  $10^0$ :**

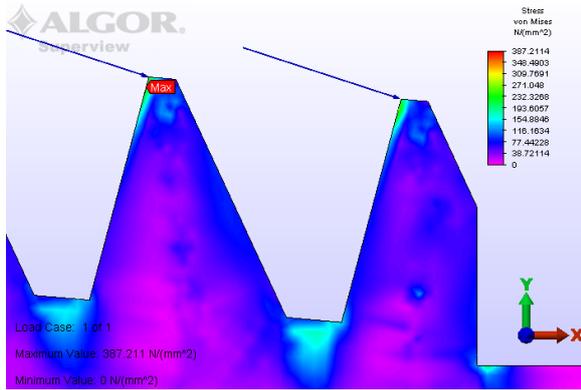
a) Entry into gear



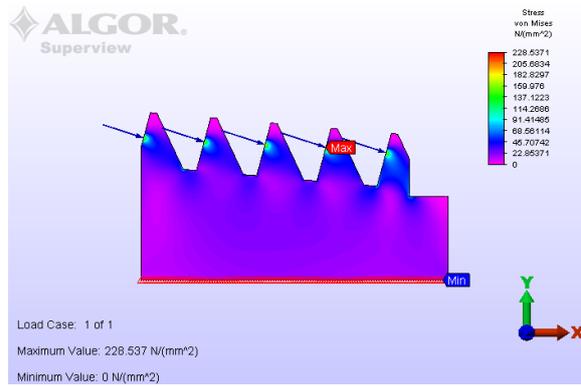
*Fig. 6. Equivalent Von Mises stress distribution*



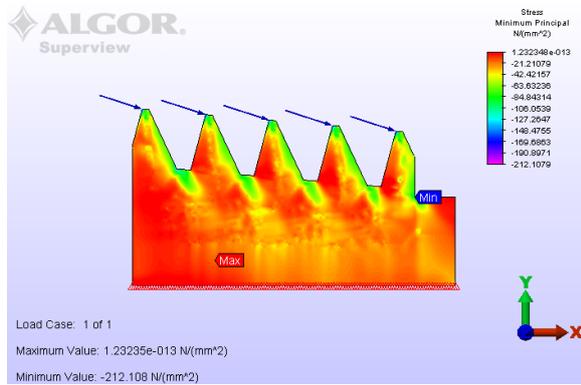
*Fig. 9. Displacement distribution – areas of equal displacement. Deformed position of the structure*



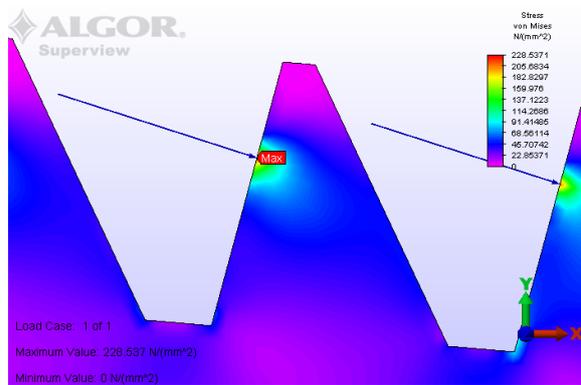
*Fig. 7. Detail – Equivalent Von Mises stress distribution in area with high application*



*Fig. 10. Equivalent Von Mises stress distribution*



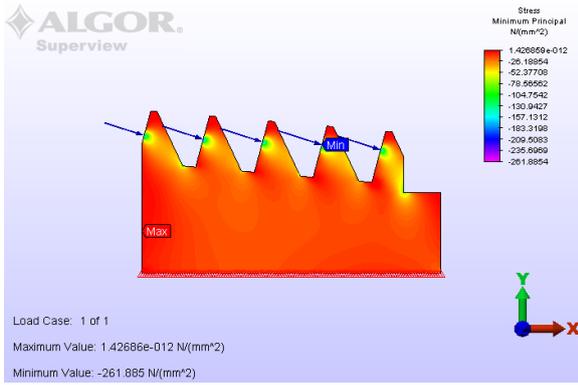
*Fig. 8. Minimum principal stress distribution (compressive stress distribution)*



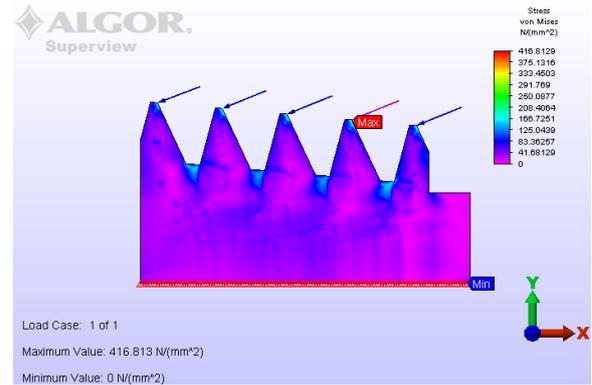
*Fig. 11. Detail – Equivalent Von Mises stress distribution in area with high application*

**Case II - load bearing flank to 30°:**

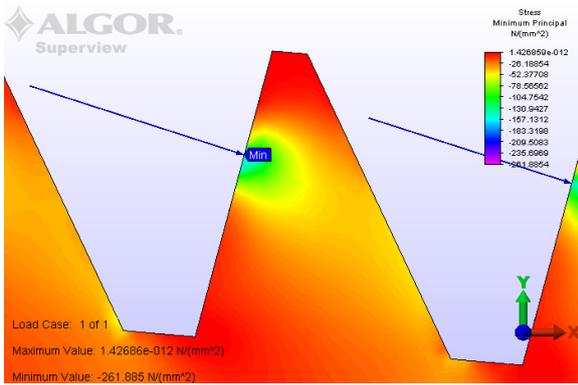
a) Entry into gear



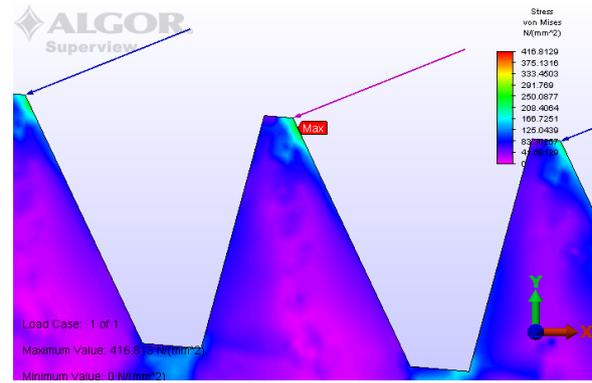
*Fig. 12. Minimum principal stress distribution (compressive stress distribution)*



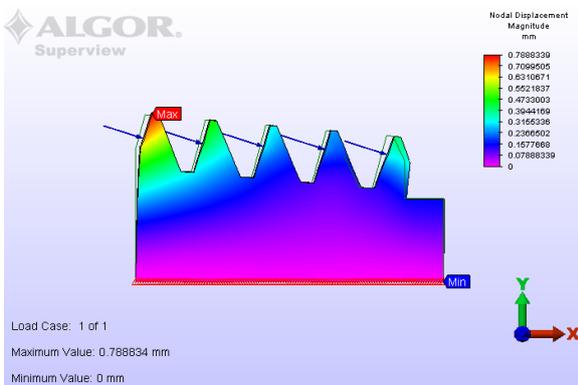
*Fig. 15. Equivalent Von Mises stress distribution*



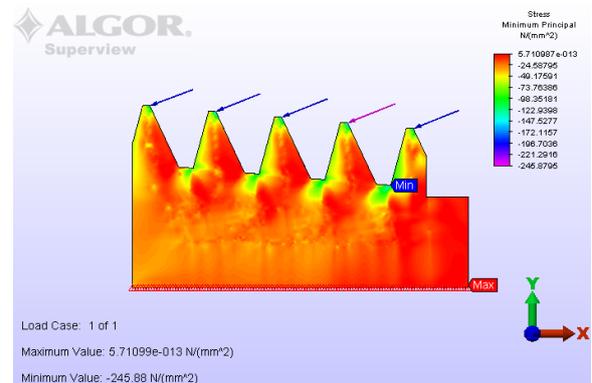
*Fig. 13. Detail – Minimum principal stress distribution in area with high application*



*Fig. 16. Detail – Equivalent Von Mises stress distribution in area with high application*



*Fig. 14. Displacement distribution – areas of equal displacement. Deformed position of the structure*



*Fig. 17. Minimum principal stress distribution (compressive stress distribution)*

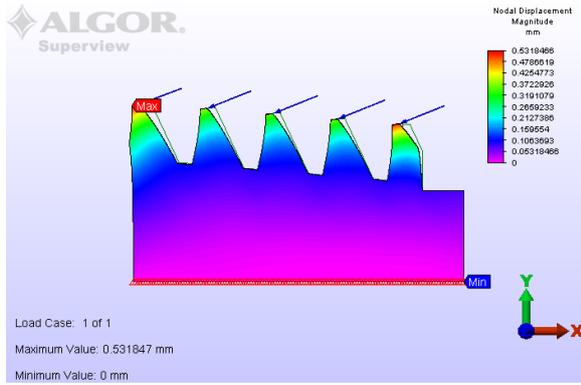


Fig. 18. Displacement distribution – areas of equal displacement. Deformed position of the structure

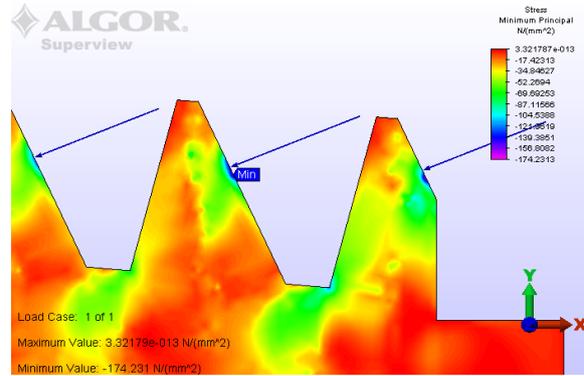


Fig. 21. Detail - Minimum principal stress distribution (compressive stress distribution)

b) Loading in operating

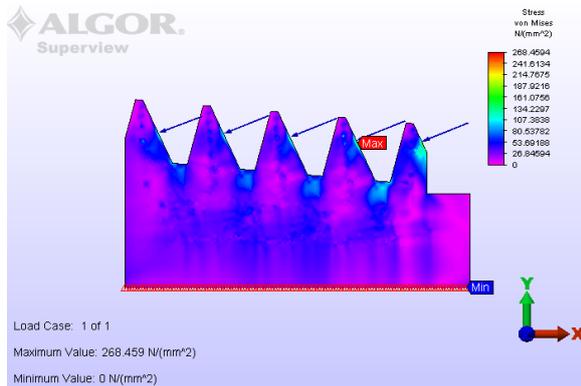


Fig. 19. Equivalent Von Mises stress distribution

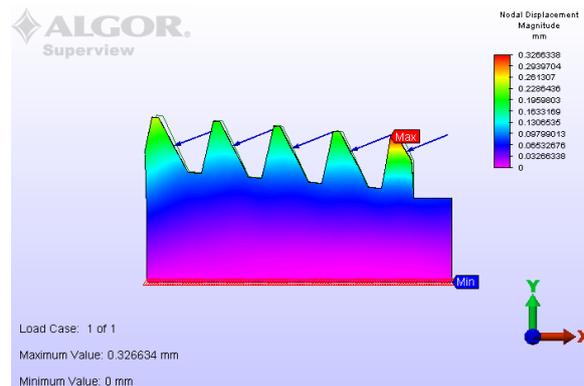


Fig. 22. Displacement distribution – areas of equal displacements. Deformed position of the structure

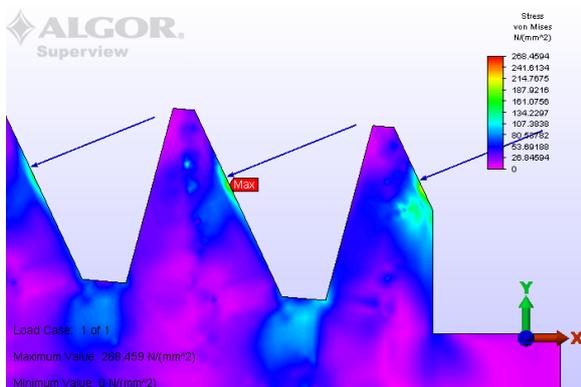


Fig. 20 Detail – Equivalent Von Mises stress distribution in area with high application

Maximum values of tension in the contact points are shown in Table 1

Table 1. Values of stresses in the contact points

Maximum stresses		Equivalent Von Mises stress $\sigma_{ech}$ [MPa]	Minimum principal stress $\sigma_2$ [MPa]
Case I Bearing flank $10^0$	a	387,211	-212,108
	b	228,537	-261,885
Case II Bearing flank $30^0$	a	416,813	-245,880
	b	268,459	-174,231

#### 4. Conclusion

Interpreting results based on simulations can be seen:

- For both cases studied, the  $10^0$  and the  $30^0$  bearing flank, the maximum stresses produced on the 4th tooth;
- Maximum stresses that occur are beyond allowable resistance of the material the worm-wheel is made of:

$$\sigma_c = 75 \text{ N/mm}^2;$$

$$\sigma_r = 100 \text{ N/mm}^2$$

- Loading on the contact point was achieved by nodal forces, maximum stresses values resulting directly being nodal point of contact;
- In fact the contact between worm-worm wheel is not as a point but is done on an area;
- The values of stresses in the immediate surrounding nodes are much smaller (Fig.23).

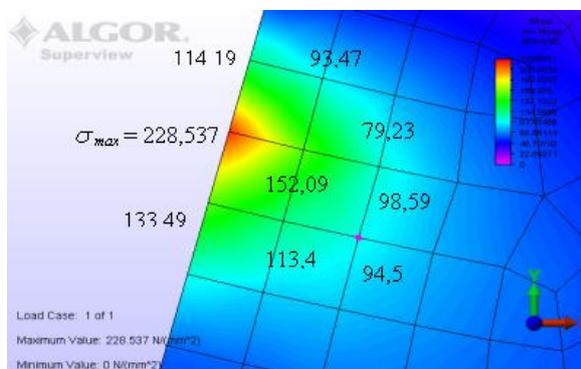


Fig. 23 Von Mises tensions surrounding nodes in contact point of worm-wheel worm

$$\sigma_{ef} \approx 150 - 200 \text{ N/mm}^2$$

- However the existing data on the calculated load, the values of power and rotation speed in operation will cause damage on 4<sup>th</sup> tooth of worm-wheel;
- In the Von Mises stress values is preponderate the compressive and stress  $\sigma$  parts (Table 1, Fig.8, Fig.12, Fig.13, Fig.17, Fig. 21);
- Move the maximum occurs when the first tooth bearing flank is  $10^0$  and the last tooth when bearing flank is  $30^0$ .

#### 5. References

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