THEORETICAL PECULIARITIES REGARDING THE DEFINITION AND REPRESENTATION OF THE ROLLING SURFACES BY A BEVEL WORM GEAR WITH THE CROWN GEAR OF INVERTED CONICITY

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ABSTRACT

This paper presents the matrix-vector calculus of the rolling surfaces of a bevel worm gear with inverted conicity. By this gear the worm gear is situated on the inner side of the crown taper. It is to remark that in this case both rolling surfaces cannot be tangent. Keeping the worm's rolling surface conical, it is necessary to calculate the profile of the crown's rolling surface that becomes a hyperboloid. Using a numerical approach, the best approximating cone of the crown wheel's rolling surface will be calculated.

Keywords: rolling surfaces, inverted bevel worm gear, numerical approach

1. Introduction

Bevel worm gears present a lot of functional advantages but- despite of this fact, they are rarely used due to the complexity of the design calculus and the difficulties characterizing the manufacturing process.

The inverted taper bevel worm gear's particularly aspect is given by the position of the worm that is situated inside the crown's taper. As a consequence, the position of the rolling surfaces is the same. The main constructive elements of such a gear are presented on figure 1.

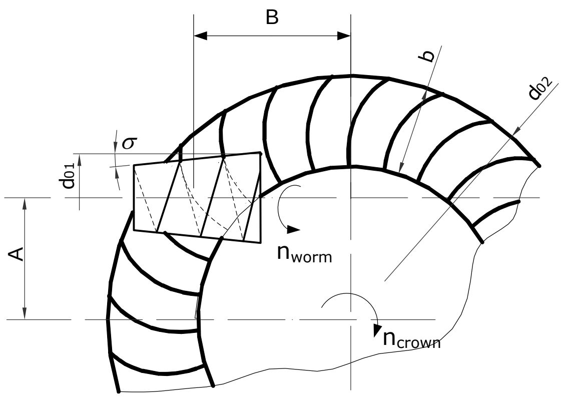

Fig. 1 Inverted taper bevel worm gear's main parameters

The geometry and the relative position of the rolling surfaces are very rarely referred in the literature.

The present paper presents a method for the calculus of the rolling surfaces of this particularly bevel worm gear. The starting hypothesis consists in keeping the conical form of the worm's rolling surface. In the relative motion, this will generate a surface family, whose meshing surface will be the theoretical rolling surface of the crown gear. Calculus of rolling surfaces is important for the future modeling of teeth form and the study of the geometrical aspects of coupling between the adjacent teeth.

2. The used coordinate systems

Figure 2 presents the relative position of the worm and the inverted tapered crown gear. Considering the relative motion of the worm to the standing crows, it executes both rotations: first about its own axis, and the second about the crown gear's axis. The S_2 system of the crown remains fixed, system S_1 of the worm is joined to the mobile system S_m. Due to the geometry of the revolved surfaces, the relative motion of the rolling surfaces is not depending on the rotation of the worm. As a conclusion, the relative motion of the rolling surfaces can be described through the motion of the S_m mobile system in the fixed S_2 system. Notations on Figure 2 are presented in Table 1.

3. The mathematical model

In order to calculate the equations of the crown gear adjacent rolling surface, the relative motion must be modeled. The parametric equations of the worm adjacent cone are the followings:

$$\begin{cases} x_1 = (r_{10} + u \operatorname{tg} \sigma_1) \cos v \\ y_1 = (r_{10} + u \operatorname{tg} \sigma_1) \sin v \\ z_1 = u \end{cases} \qquad (1)$$

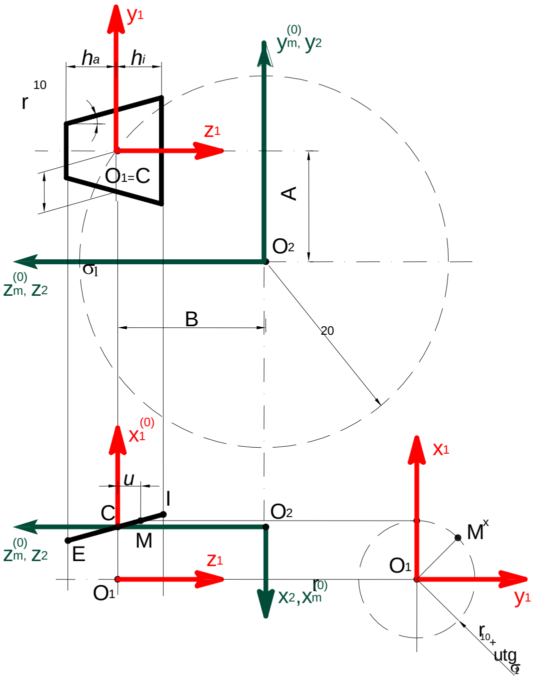

Fig. 2 Definition of the coordinate systems

where $u \in [-h_a, h_i]$, $v \in [0, 2\pi]$

Table 1. Notations defined

A	Axis distance
B	"C" contact point reference distance
σ_1	Worm taper angle
r_{10}	Taper reference radius corresponding to the contact point "C"
r_{20}	Crown reference radius, corresponding to contact point "C"
h_a, h_i	Axial distance parameters localizing the contact point "C" on the worm's taper.
u,v	Surface parameters of the worm's taper
S1	$O_1X_1Y_1Z_1$ – worm gear's system
S2	$O_2X_2Y_2Z_2$ – crown gear's system
S3	$O_2X_mY_mZ_m$ – mobile system

Systems S_1 and S_M are bounded by a translation described by the following equation in homogenous coordinates:

$$\begin{pmatrix} x_m \\ y_m \\ z_m \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & r_{10} \\ 0 & 1 & 0 & A \\ 0 & 0 & -1 & B \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} \qquad (2)$$

The relative rotation of the worm about the crown's axis is described by the following matrix equation

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_m \\ y_m \\ z_m \\ 1 \end{pmatrix} \qquad (3)$$

Using equations (1) and performing the calculus given (2) and (3) result the equations of the surface family generated by the worm in its relative motion:

$$\begin{cases} x_2(u,v,\varphi) = -(r_{10} + u \operatorname{tg} \sigma_1)\cos v + r_{10} \\ y_2(u,v,\varphi) = [(r_{10} + u \operatorname{tg} \sigma_1)\sin v + B]\cos\varphi - \\ \quad -(-u+B)\sin\varphi \\ z_2(u,v,\varphi) = [(r_{10} + u \operatorname{tg} \sigma_1)\sin v + B]\sin\varphi + \\ \quad +(-u+B)\cos\varphi \end{cases} \qquad (4)$$

The calculus of the meshing surface is quiet simple due to the fact that generating surfaces are revolved. In this case the condition of meshing can be primed through the intersection of the normal of the generating surface with the axis of rotation of the enveloping surface (the crown related surface in this case). The problem will be solved in the system S_2, Performing the matrix product that follows from (2) and (3) results:

$$\mathbf{M}_{21} = \mathbf{M}_{2m}\mathbf{M}_{m1} = \\ = \begin{bmatrix} -1 & 0 & 0 & r_{10} \\ 0 & \cos\varphi & \sin\varphi & A\cos\varphi - B\sin\varphi \\ 0 & \sin\varphi & -\cos\varphi & A\cos\varphi + B\sin\varphi \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

The definition of the normal vector of the generating bevel surface is in conformity with Figure 3. It can be written that

$$\mathbf{n}_1 = \begin{bmatrix} \cos\sigma_1 \cos v \\ \cos\sigma_1 \sin v \\ -\sin\sigma_1 \end{bmatrix} \qquad (6)$$

Using the rotation part of the matrix (5), the above normal vector in the system of the crown will get the form

$$\mathbf{n}(u,v,\varphi)=\mathbf{L}_{21}\,\mathbf{n}_1(u,v)=$$

$$=\begin{bmatrix}-\cos\sigma_1\cos v\\ \cos\sigma_1\sin v\cos\varphi-\sin\sigma_1\sin\varphi\\ \cos\sigma_1\sin v\sin\varphi+\sin\sigma_1\cos\varphi\end{bmatrix} \quad (7)$$

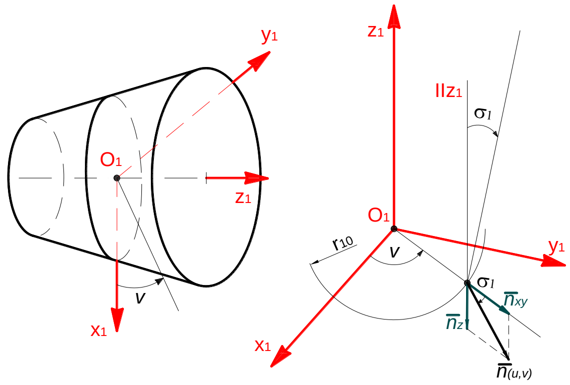

Fig. 3 Definition of the cone's normal vector in its own coordinate system

The equation of the normal line is:

$$\frac{x_2-x_2(u,v,\varphi)}{n_{x2}}=\frac{y_2-y_2(u,v,\varphi)}{n_{y2}}=$$
$$=\frac{z_2-z_2(u,v,\varphi)}{n_{z2}}=\lambda \quad (8)$$

Let's denote in order to simplify some expressions as follows:

$$E_1=x_2(u,v,\varphi)n_{y2}-y_2(u,v,\varphi)n_{x2}$$
$$E_2=x_2(u,v,\varphi)n_{z2}-z_2(u,v,\varphi)n_{x2} \quad (9)$$

Now, the condition of intersection of the normal line and the crown's axis can be written as the four equations containing linear system by three unknowns formed by the equations of the normal and the crown's axis must be compatible. The main determinant of

$$\begin{cases}x_2n_{y2}-y_2n_{x2}=E_1\\ x_2n_{z2}-z_2n_{x2}=E_2\\ y_2=0\\ z_2=0\end{cases} \quad (10)$$

must be zero, thus:

$$\begin{vmatrix}n_{y2} & -n_{x2} & 0 & E_1\\ n_{z2} & 0 & -n_{x2} & E_2\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{vmatrix}=0 \quad (11)$$

Developing the calculus on equation (11) the condition of enveloping becomes:

$$n_{x2}(n_{z2}\,y_2(u,v,\varphi)-n_{y2}\,z_2(u,v,\varphi))=0 \quad (12)$$

Finalizing the calculus and replacing the function $\varphi(u,v)$ in equations (4), the enveloping surface meshed by the cone family will be the following:

$$\begin{cases}x_2(u,v)=-\dfrac{Q-B}{\sin v}\cdot\cos v+r_{10}\\ y_2(u,v)=Q\cos\varphi-R\sin\varphi\\ z_2(u,v)=Q\sin\varphi+R\cos\varphi\end{cases} \quad (13)$$

Here Q and R denote some expressions depending on u:

$$Q=(r_{10}+u\,\mathrm{tg}\,\sigma_1)\sin v+B$$
$$R=-u+B \quad (14)$$

4. Calculus of the contact curve

The shape of the contact curve between one cone of the surface family described by the worm associated rolling surface in the S_2 crown's system remains constant during the meshing process, due to the fact that both surfaces are revolved and the relative motion has one axis of symmetry.

If solving equation (12) for the peculiar value of motion parameter $\varphi=0$, the function $\varphi(u,v)$ will reduce to a $v(u)$ form that implemented in parametric equations (13) give the equations of the contact curve, relative to the S_2 coordinate system of the crown. This corresponds to the starting position of the worm-associated cone.

The axial profile of the meshed surface representing the crown's associated rolling surface is a flat curve. It can be obtained through $z_2(u,v)=0$, obtaining from this another $v(u)$ function and implementing this in the first two equations of (13). But, the contact curve once determined, it is easier to write for each $x_2(u,v(u))$ the distance of the considered surface point to the x_2 axis of revolution. Following this procedure, the axial profile of the crown's rolling surface can be written as

$$x_{ax}=x_2(u,(v(u))$$
$$y_{ax}=\sqrt{[y_2(u,(v(u))]^2+[z_2(u,(v(u))]^2} \quad (15)$$

5. Numerical approach of best approximating cone surface

The best approximating cone of the revolved hyperboloid (13) serves for dimensioning calculus and estimation of tooth dimensions. In the following the best approximating cone's taper angle will be determined, using the least squares method. If considering the equation of generatrix given by $y_2=px_2+q$ and the distance from a given profile point to this by $\delta_i = y - y_i = px_i + q - y_i$, p and q result from the following system of equations:

$$\begin{cases} \Phi(p,q) = \sum_{i=1}^{n} \delta_i^{\,2} = \sum_{i=1}^{n} (px_i + q - y_i) \\ \dfrac{\partial \Phi}{\partial p} = 0 \\ \dfrac{\partial \Phi}{\partial q} = 0 \end{cases} \qquad (16)$$

Here n denotes the length of the set of points.

6. Aplication

An inverted tapered worm bevel gear is considered here having the following main dimensions:

- worm reference radius r_{12}=13mm;
- worm taper half angle: σ_1=5^0;
- crown wheel reference radius: r_{20}=75mm;
- axis distance: A_w = 58mm
- contact point distance : B_w = 47,55mm
- inner axis distance on worm taper h_i=14mm;
- outer axis distance on worm taper h_a=14mm;

Following the calculus presented above the results obtained are the followings.

Figure 4 shows the shape of the $v(u)$ function used for calculus of axial profile (15).

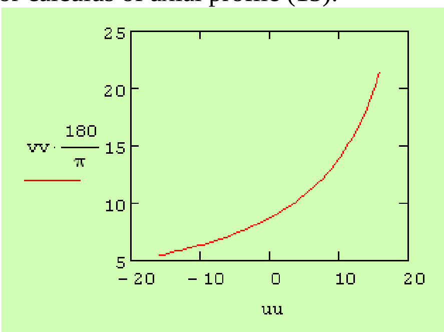

Fig. 4 Shape of the $v(u)$ function

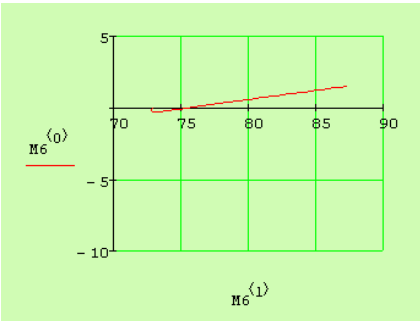

Fig. 5 The best approximating generatrix

The best approximating taper's generatrix is presented in figure 5. Figures 6 and 7 shown the rolling surfaces determined as reciprocal enveloping surfaces. Let's remark the shadow-like appearance of the contact curve on figure 7.

7. Conclusions

On the method presented in the precedent sections it can put the following conclusions:

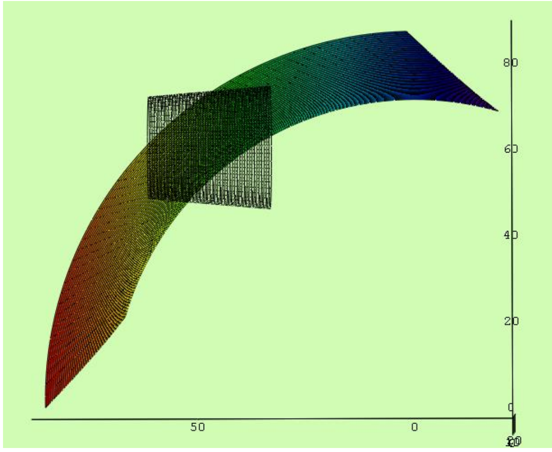

Fig. 6 The reciprocal enveloping rolling surfaces of the worm and the crown wheel (upper sight)

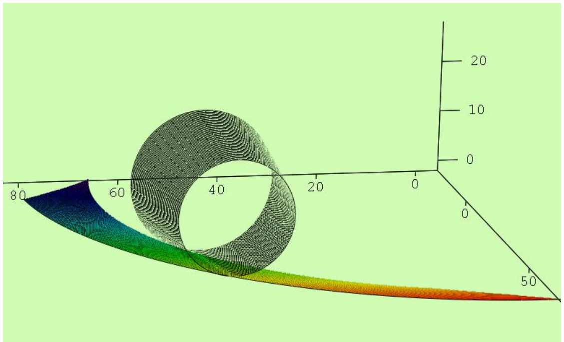

Fig. 7 The reciprocal enveloping rolling surfaces of the worm and the crown wheel (upper sight)

- The calculus and representation of the rolling surfaces is the first step in designing inverted taper worm bevel gears. Classical worm bevel gears accept two cones as rolling surfaces because they can have external tangentia. Well, that's impossible in the case presented above.
- This calculus returns results that are indispensable for determnning the form of teeth and avoiding interference.
- It is the only accessible way to determine the necessary geometrical elements.

8. References

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