

# $\begin{array}{l} \textbf{ROBUST} \ \textbf{H}_{\infty} \ \textbf{POSITION} \ \textbf{CONTROL} \ \textbf{OF} \ \textbf{AN} \ \textbf{UNLOADING} \\ \textbf{MACHINE} \ \textbf{FROM} \ \textbf{A} \ \textbf{ROTARY} \ \textbf{HEARTH} \ \textbf{FURNACE} \\ \end{array}$

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## ABSTRACT

This paper presents the design of a sub-optimal  $H_{\infty}$  robust controller for the position control of an unloading machine, from a rotary hearth furnace. In the modeling of the positioning system, the mass of the unloading machine was taken as an approximation of the real value, introducing uncertainties in the position control. In this case, a robust controller was designed using multiplicative uncertainties in order to improve the unloading rate of the furnace. The plant behavior for the nominal case, the perturbed parameters cases and the robustness of the controller is validated by simulation.

Keywords: Rotary hearth furnace, unloading machine, positioning system, robust control, suboptimal  $H_{\infty}$  control

### 1. Introduction

The rotary hearth furnace is the first important aggregate in the technological flow of the hot rolling process [1]. The main role of this furnace is to heat up blocks of billets from the ambient temperature to the rolling temperature, which is about  $1250^{\circ}C$  [2]. The furnace consists of five regulating temperature sectors, a sector for preheating, and a sector for charging and discharging billets [3]. For the loading and unloading process the furnace is equipped with a loading machine and an unloading one. The furnace's hearth is rotated and its rotation regime can be jerky with stops, at fixed or continuous angles. When the hearth stops the machines load, respectively unload, the block of billets from the furnace.



Fig. 1 – The unloading machine representation

The unloading or the loading machine from Tenaris Silcotub Zalau is essentially a trolley that moves on a rail way track. The fig. 1 shows the schematic representation of the unloading machine that has a long arm with a clamp at the furnace end. The clamp is design to catch the billet right in the middle according to its length and it is designed to have one fixed and one movable jaw. The robotic arm has to be positioned correctly over a billet in order for the clamp to catch it [4]. To ensure the entrance of the clamp jaws the minimum distance between two adjacent billets inside the furnace is 100 mm, and in addition to avoid deteriorating the billets.

The distance between the initial position of the machine and the position of the billet on to the furnace hearth is known according to the loading scheme. An incremental encoder is used in order to measure the machine's distance from the initial position to the billet's position inside de furnace.

An issue that appears during the billets unloading process is that a very precise brake is needed for the rotary hearth in order to stop the billet exactly at the position of the unloading rail way axis. Because of this, at the outlet door of the furnace, the billet has a small displacement from the rail way axis. In order to positionate the clamp over the billet, the unloading arm is able to rotate around a static point, allowing a horizontal movement of 300 - 400 mm. Since the distance obtained by the rotation of the arm is small compared to the billets length, the motion is approximated by a linear movement.

For obtaining a correct positioning of the clamp over the unloading billet, an automated control system is used to control the horizontal unloading arm movement. The position control system, implemented at Tenaris Silcotub Zalau uses a PID controller for the unloading arm movement. The current control system performances are an overshoot of 11% and a settling time of 2 seconds.

The goal of this paper is to improve the unloading rate of the furnace, which means decreasing the current settling time, by designing a sub – optimal  $H_{\infty}$  controller. To the authors best knowledge we are the first to treat this issue.

# 2. Mathematical model for the unloading machine's positioning system

The horizontal positioning system for the unloading arm consists of a cylinder which is controlled using a proportional directional valve.

The electro-hydraulic proportional valve used is manufactured by Bosch Rexroth, and it consists in a directly controlled valve with integrated control electronics. The nominal working flow amounts to 75 l/min at 10 bar valve pressure differential [5]. The hydraulic cylinder is manufactured by Parker Hannif Corporation and it is a type MP3 from MMA duty series. It was designed for service in steel mills and it has a a minimum stroke of 25 mm and a maximum stroke of 150 mm [6].

The position of the cylinder is measured with a linear transducer. The measurement signal is used to determine the displacement of the arm from the rail way axis in milimeters unit. The linear encoder is manufactured by Gefran and it can measure displacement or speed. The sampling time of the typical position read is 1 ms [7].

Due to constituent elements of the positioning system for the unloading arm, the system is considered to be an electro-hydraulic axis and for consideration of time constants we take into account only the dynamics of the hydraulic part, through the main equations described below [8].

Linear equation of servo valves:

$$\Delta Q = K_Q \Delta x - K_C \Delta p_m \tag{1}$$

Equation of flow conservation:

$$\Delta Q = S \frac{d(\Delta y)}{dt} + \alpha \Delta p_m + \frac{V_T}{4E} \frac{d(\Delta p_m)}{dt}$$
(2)

Mechanical equation of motion:

$$S\Delta p_m = m \frac{d^2(\Delta y)}{dt^2} + f \frac{d(\Delta y)}{dt} + F_R$$
(3)

Where:  $K_Q$  – flow gain,  $K_c$  – flow-pressure coefficient of the proportional valve,  $\Delta Q$  – flow differential,  $\Delta x$  – billet displacement,  $\Delta p_m$  – pressure differential, S – piston area,  $\alpha$  – overall rate of oil loss,  $V_T$  – total oil volume,  $\Delta y$  – output size (arm displacement), E – coefficient of oil elasticity, m – mass of the piston and the load, f – viscous damping coefficient,  $F_R$  – static force strength.

By applying the Laplace transformation to (1) - (3), and by neglecting the parameters a,  $K_c$ , and  $F_R$  the mathematical model (4) was obtained:

$$\Delta y = \frac{k\omega^2}{s^2 + 2\xi\omega s + \omega^2} \Delta x \tag{4}$$

From (4) the state space representation was obtained:

$$\begin{aligned}
\mathbf{x}_1 &= \omega(u - x_2) \\
\mathbf{x}_2 &= \omega(x_1 - 2\xi x_2) \\
\mathbf{y} &= kx_2
\end{aligned}$$
(5)

The mathematical and the state space model parameters are defined as follows:

$$k = \frac{K_{\varrho}}{S}$$

$$\omega = \sqrt{\frac{4ES^2}{V_T m}}$$

$$\xi = \frac{V_T f}{8ES^2} \cdot \sqrt{\frac{4ES^2}{V_T m}}$$
(6)

The mass of the unloading machine was calculated by multiplying its density with an approximated volume. The state space parameters were obtained by replacing the parameters with the values from the data sheets and considering the aproximated value of the mass. The following values were obtained.

$$k = 13.281$$
  

$$\omega = 797.1495$$
  

$$\xi = 3.0355 \cdot 10^{-7}$$
(7)

# 3. $H_{\infty}$ robust control design

The main goal of a robust control is designing a controller that stabilises the process not only for its nominal parameters values, but for the case in which the system parameters vary within certain limits also. The controller and the controlled plant must satisfy certain performance requirements like: low overshoot, short settling time and also disturbance rejection. In order to satisfy the above requirement we designed and simulated a H<sub>∞</sub> robust control.

The robust  $H_{\infty}$  otpimization investigation began with the consideration of minimizing the  $\infty$ norm of the sensitivity function of a single-input single-output linear feedback system [9]. It was soon extended to more general problems, when the stability robustness criterion confirmed the relevance of the  $\infty$ -norm for robustness [10]. The  $H_{\infty}$  controller is an effective and efficient robust design method for linear systems and it ensures robust performance in response to both external disturbances and parameter uncertainty [11].

The H<sub> $\infty$ </sub> sub-optimal controller design consists in finding a controller K, for which the H<sub> $\infty$ </sub> norm of the closed loop transfer function will be less than a given positive number [12]. The controller has to ensure robust stability for the closed loop system, good tracking, attenuate the influence of the exogenous input signals on the controller signal and to reject the noise [13]. The H<sub> $\infty$ </sub> robust control design consists in finding a controller that minimize the lower linear fractional transformation (LFT) for both the plant, and the controller F(G,K) [14].

$$J_{\infty}(K) = \left\| F(G, K) \right\|_{\infty} < \gamma \tag{8}$$

In order to design the robust controller the augmented plant mathematical model (G) described by the following state space equations has to be constructed.

$$\dot{x}(t) = Ax(t) + B_1v(t) + B_2u(t)$$
  

$$z(t) = C_1x(t) + D_{11}v(t) + D_{12}u(t) \qquad (9)$$
  

$$y(t) = C_2x(t) + D_{21}v(t) + D_{22}u(t)$$

Here x is the state variable vector, v the exogenous inputs vector, u is the control input vector, z is the vector of the signals that are considered to be important for the closed-loop performance, while y is the vector of measurements. The matrix form of the augmented plant model is:

$$G = \begin{pmatrix} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{pmatrix}$$
(10)

In order to obtain better performances for the closed-loop system weighting functions (W) can be added:

$$\min_{Kstab} \left\| \frac{W_1 (I + GK)^{-1}}{W_2 K (I + GK)^{-1}} \right\|$$
(11)

The sensitivity function S (12) characterizez the sensitivity of the control system output to disturbances. To limit the control signal the KS sensitivity was introduced.

$$S = (I + GK)^{-1}$$
(12)

4. Robust control for the unloading positioning system

The  $H_{\infty}$  suboptimal controller design begins with the state-space representation of the positioning system for the unloading machine describe by (5). Figure 2 shows the block representation for the state space system's model.



### Fig. 2 - Block representation for the system model

The model parameters (7) are considered to be the nominal values for the state space model of the unloading arm positioning system. It was considered that the mass can vary  $\pm 20\%$  and, thus, the model parameters  $\omega$  and  $\zeta$  can vary up to  $\pm 10\%$ . The uncertain parameters  $\omega$  and  $\xi$  may be represented as follows:

$$\omega = \omega_N (1 + p_\omega \delta_\omega)$$
  

$$\xi = \xi_N (1 + p_\xi \delta_\xi)$$
(13)

where  $\omega_N$ ,  $\xi_N$  are the nominal values that can be found in (7), while  $p_{\omega}$ ,  $p_{\xi}$  are the maximum relative uncertainties, and  $\delta_{\omega}$ ,  $\delta_{\xi} \in [-1,1]$  being the relative variations, which means that the parameters  $\omega$  and  $\xi$  vary by ±10%. The parameters  $\omega$  and  $\xi$  may be represented as a lower LFT:

$$\begin{split} & \omega = \mathcal{F}_{L}(M_{\omega}, \mathcal{O}_{\omega}) \\ & \xi = \mathcal{F}_{L}(M_{\xi}, \mathcal{O}_{\xi}) \end{split} \tag{14}$$

where

$$\boldsymbol{M}_{\boldsymbol{\omega}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{\omega}_{N} \\ \boldsymbol{p}_{\boldsymbol{\omega}} & \boldsymbol{\omega}_{N} \end{bmatrix}, \boldsymbol{M}_{\boldsymbol{\xi}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{\xi}_{N} \\ \boldsymbol{p}_{\boldsymbol{\xi}} & \boldsymbol{\xi}_{N} \end{bmatrix}$$
(15)

For the uncertain parameters blocks containing the parameter  $\omega$ , or the parameter  $\xi$ , from fig. 2, will be replaced with the group of block like the ones in fig. 3 or in fig. 4.



Fig. 3 - Block for ω parameter with uncertainties



Fig. 4 - Block for  $\xi$  parameter with uncertainties

The equations coresponding to the blocks in fig. 3 and in fig. 4 are:

$$v_{\omega 1} = p_{\omega} u_{\omega 1} + \omega_N u$$
  
$$v_{\omega 1} = \omega_N u$$
 (16)

$$v_{\xi 1} = \xi_N v_{\omega 3} + p_{\xi} u_{\xi 1}$$
(17)

$$y_{\xi 1} = \xi_N v_{\omega 3}$$

By replacing each parameter block from fig. 2, we obtain five exogenous inputs:  $u_{\omega l}$ ,  $u_{\omega 2}$ ,  $u_{\omega 3}$ ,  $u_{\zeta l}$ ,  $u_{\omega 4}$ , and five important signals:  $y_{\omega l}$ ,  $y_{\omega 2}$ ,  $y_{\omega 3}$ ,  $y_{\zeta l}$ ,  $y_{\omega 4}$ .

The equations that governs the dynamic behaviour of the system with perturbed parameters are:

$$\dot{x}_{1} = v_{\omega 1} + v_{\omega 4} = -\omega_{N}x_{2} + p_{\omega}u_{\omega 1} + p_{\omega}u_{\omega 4} + \omega_{N}u$$
$$\dot{x}_{2} = v_{\omega 2} + v_{\xi 1} = \omega_{N}x_{1} - 2\xi_{N}\omega_{N}x_{2} + p_{\omega}u_{\omega 2}$$
$$+ \xi_{N}p_{\omega}u_{\omega 3} + p_{\xi}u_{\xi 1}$$
$$v_{\lambda} = \omega_{N}u$$

$$y_{\omega 2} = \omega_N x_1$$

$$y_{\omega 3} = -2\omega_N x_2$$

$$y_{\xi 1} = \xi_N v_{\omega 3} = -2\xi_N \omega_N x_2 + \xi_N p_\omega u_{\omega 3}$$

$$y_{\omega 4} = -\omega_N x_2$$

$$y = kx_2$$

$$(18)$$

From the above equations the augmented plan model G is obtained, and it coresponds to the state space representation (9).

	0	$-\omega_N$	$p_{\omega}$	0	0	0	$p_{\omega}$	$\omega_N$	
<i>G</i> =	$\omega_{N}$	$-2\xi_N\omega_N$	0	$p_{\omega}$	$\xi_N p_\omega$	$p_{\xi}$	0	0	
	0	0	0	0	0	0	0	$\omega_N$	
	$\omega_{N}$	0	0	0	0	0	0	0	(19)
	0	$-2\omega_N$	0	0	0	0	0	0	
	0	$-2\xi_N\omega_N$	0	0	$\xi_N p_{\omega}$	0	0	0	
	0	$-\omega_{N}$	0	0	0	0	0	0	
	0	k	0	0	0	0	0	0	

In order to ensure good disturbance attenuation and good transient response, weighting functions were added.

The following two weighting functions were obtained iteratively, starting with the weighting functions [15]:

$$W_{p} = 0.3 \frac{s^{2} + 5s + 10}{s^{2} + 2s + 0.0001}$$
(20)  
$$W_{\mu} = 0.02$$

The coefficients were adjusted until the systems transient response for the nominal values had the required performances.

The transformation from the state-space representation to the transfer function representation is given by:

$$H_f(s) = C(sI - A)^{-1}B$$
 (21)

Considering the plant agumented model (19), the weighting functions (20), and the transformation (21), we obtained the  $H_{\infty}$  sub-optimal controller:

$$H_{\kappa}(s) = \frac{47.18s^3 + 157.3s^2 + 2.998 \cdot 10^7 s + 9.991 \cdot 10^7}{s^4 + 1.626 \cdot 10^4 s^3 + 1.33 \cdot 10^8 s^2 + 2.655 \cdot 10^8 s + 1.327 \cdot 10^4} (22)$$

# 5. Simulation results for the closed-loop position control system

The  $H_{\infty}$  suboptimal controller was determined using the Robust Control Toolbox within MATLAB

7.6 [16]. The system's outputs were also ploted using MATLAB, by simulating the closed-loop position control system that has on the direct path the  $H_{\infty}$  sub-optimal controller and the state space mathematical model for the unloading machine.

For the assessments of the performance of the closed-loop robust positioning system for the nominal case, the step response for a 1 mm billet displacement was simulated as shown in fig. 5.



The settling time for the position control system is aproximately 1.6 s, compared with the 2 s settling time in the case of the current PID position control system. The overshoot is 5%, lower than the current one of 11%. The closed loop system has no stationary error.

Figure 6 shows the step responses for the perturbed and for the nominal system. It can be seen the overlapping of the responses for the  $\pm 10\%$  perturbed system,  $\pm 5\%$  perturbed system, and the nominal system step response. We applied the 8 mm positive reference and the -6 mm negative reference which can be related to a billet displacement to the right of the rail way axis for the positive reference, and a billet displacement to the left of the rail way axis, for the negative setpoint.

The performances for the closed-loop positioning system remain the same even if the machine's mass varies up to  $\pm 20\%$ .

In order to demonstrate the robust performances in response to external disturbaces the  $\pm 10\%$  perturbed system, the  $\pm 5\%$  perturbed system outputs for a 2 mm step disturbance are presented in fig. 7. The robust positioning controller rejects the disturbance in about 1.5 s.



Fig. 6 - Closed loop step responses for the nominal and the perturbed systems



Figure 8 shows the inverse of performance wieghting function and the nominal sensitivity plot. From this plot we observe that for low frequencies the closed-loop system must attenuate the output disturbance. The stability of the nominal, and the perturbed, system is proved by the sensitivity function of the closed-loop system being lower than the inverse of the weighting function



### Fig. 8 - Inverse of performance weighting function

#### 6. Conclusion

In this paper, the control of a horizontal positioning system for an unloadin machine with uncertainties in parameters is considered. In the simulation studies of the system dynamic behaviour the state space mathematical model of the system has been used. For the task of designing a robust  $H_{\infty}$  suboptimal controller an uncertain model has been derived from the state space position model.

The application of linear robust control design techniques on an unloading machine positioning system resulted in a controller that is able accordind to simulation to decrease the settling time from 2 s to 1.6 s and to improve the billets unloading rate from the furnace with 20%. The controller is also able to decrease the system's overshoot by approximately 50%.

The robustness of the controller designs is validated by simulation. The resulting robust controller guarantees stability and better performances for the closed-loop system.

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