HILBERT TRANSFORM ALGORITHM IN LABVIEW

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Abstract

This paper presents an alternative algorithm for Hilbert transform calculation. This algorithm was implemented in LabVIEW software. It was tested for some different elementary signals. The results were compared with the method proposed by LabVIEW programming environment of National Instruments Company.

Keywords: Hilbert transform, Fourier transform, analytical signal.

1. Introduction

It is known that the Discrete Fourier transform is the mathematical tool used in signal processing and analysis.

Also, in programming environments (such as Matlab, Mathcad, LabView, C + +), the procedures for calculating the Discrete Fourier transform are presented by various algorithms.

This paper proposes an algorithm for computing the Discrete Hilbert transform, starting from known methods for calculating the discrete Fourier transform.

With the method proposed in the paper, the user of a particular programming environment can quickly implement the algorithm of Discrete Hilbert transform.

Hilbert transform is a linear operator, introduced in 1905 by David Hilbert. It is used in mathematics and signal processing. Although it was discovered more than a century ago, it was left over a while.

With the development produced in the field of computer technology, it was possible to board with success the Hilbert transform.

Hilbert transform has shown its importance not only as a mathematical tool but also in engineering. There are multiple applications of Hilbert transform in electrical drives, automotive, geophysical, multimedia, radio and television.

Hilbert transform is used in signal theory for analytical signals synthesis in the modulation process.

In Electrical Engineering, Hilbert transform is well known because it manages to do the $\pm \pi/2$ phase shift existing among sinusoidal signals.

An important aspect about Hilbert transform is that it keeps the original signal domain.

This present article proposes to develop a time domain algorithm calculation of Hilbert transform. It also proposed to study the algorithm behavior for different input signals.

In the beginning part of this paper, are presented theoretical considerations of elementary signals and the transition from time to frequency domain.

Then, it is set out the Hilbert transform both in time domain and frequency domain. Hilbert transform of a real signal is introduced by convolution, as an imaginary part of the analytical signal and by $\pm\pi/2$ phase shift.

After the Hilbert transform algorithm is developed, it is implemented in LabVIEW environment. The procedure starts with a harmonic signal. This signal is sampled and then it is passed from time domain to frequency domain. The signal supports some mathematical transformation before it returns to time domain. Thus is obtained an algorithm for computing the Hilbert transform in time domain.

As well, this paper will highlight the Hilbert transform for different input signals. ([3]-[7])

2. General presentation

There is information that cannot be highlighted in time domain signals. For those signals it is used frequency domain. With Fourier transform it is possible to pass a signal from time domain to frequency domain.

- *Continuous Fourier transform:*

$$X(j\omega) = TF\{x(t)\} = \int_{-T}^{T} x(t)e^{-j\omega t}dt \qquad (1)$$

$$x(t) = TF^{-1}\{X(j\omega)\} = $$
$$= \frac{1}{T}\int_{-T}^{T} X(j\omega)e^{j\omega t}d\omega \qquad (2)$$

A discrete signal is given by a vector of data acquired from a time continuous signal. A time continuous signal is called analog. A discrete signal is

called digital.

Let there be $x(t)$ a time continuous signal. $x[n]$ is obtained by time discretization of $x(t)$ with Δt. Where Δt is the sampling step.

$$t = n\Delta t, \ x[n\Delta t] \Rightarrow x[n]$$
$$\omega = k\Delta\omega, \ X[jk\Delta\omega] \Rightarrow X[k], \ \int \Rightarrow \sum$$

- *Discrete Fourier transform:*

$$X[k] = TFD\{x[n]\} = \sum_{k=0}^{N-1} x[n]e^{-jkn\frac{2\pi}{N}} \tag{3}$$

$$X[k] = TFD^{-1}\{X[k]\} = \frac{1}{N}\sum_{n=0}^{N-1} X[k]e^{jkn\frac{2\pi}{N}} \tag{4}$$

- *Elementary signals:*

This article uses the following signals: sine wave, square wave, triangle wave and sawtooth wave.

- *Hilbert transform. Definitions. Properties*

For $x(t)$, Hilbert transform produces a $H\{x(t)\}$ function. In signal processing it is found like " $\hat{x}(t)$ ".

$$H(x(t)) = \hat{x}(t) = [x(t)]^{\wedge} \tag{5}$$

where $\hat{x}(t)$ is called the conjugate of $x(t)$.

a. Hilbert transform defined by the convolution product

Hilbert transform represents in fact the convolution between $x(t)$ and $\frac{1}{\pi t}$.

$$H\{x(t)\} = x(t) * \frac{1}{\pi t} \tag{6}$$

$$\hat{x}(t) = \frac{1}{\pi} \, v.p. \int_{-\infty}^{\infty} x(\tau)\frac{1}{t-\tau} d\tau \tag{7}$$

$$\hat{x}(t) = \frac{1}{\pi}[\lim_{\varepsilon \to 0} \int_{-\infty}^{-\varepsilon} x(t-\tau)\frac{1}{t} d\tau +$$
$$+ \lim_{\varepsilon \to 0} \int_{+\varepsilon}^{\infty} x(t-\tau)\frac{1}{t} d\tau] \tag{8}$$

The negative form of the original signal can be obtained if Hilbert transform it is applied twice:

$$[\hat{x}(t)]^{\wedge} = -x(t) = H^{-1}\{\hat{x}(t)\} \tag{9}$$

The Hilbert transform used four times on the same real signal gives us the original function back.

b. Hilbert transform defined as imaginary part of an analytical signal

A Hilbert pair is formed by $x(t)$ signal together with the $\hat{x}(t)$ signal.

An analytical signal is a complex signal. The real part is given by $x(t)$ and the imaginary part is given by $\hat{x}(t)$.

Let there be the analytical signal $z(t)$, then:

$$z(t) = x(t) + j\hat{x}(t) \tag{10}$$

The module of the complex signal $z(t)$ is:

$$|z(t)| = \sqrt{x^2(t) + \hat{x}^2(t)} \tag{11}$$

and it is called "analytical signal envelope", and the phase of the complex signal is:

$$\arg z(t) = \tan^{-1}\frac{\hat{x}(t)}{x(t)} \tag{12}$$

$$H\{x(t)\} = Im\{z(t)\} \tag{13}$$

c. Hilbert transform defined by phase shift:

By applying the Hilbert transform on a signal under harmonic form, each cosine harmonic becomes a spectral component with the same magnitude and with $(-\frac{\pi}{2})$ phase shift.

This method can be applied on periodic signals. For example it is given the signal $\cos(\omega_0 t)$ and Hilbert transform is calculated as follows:

$$H\{\cos(\omega_0 t)\} = \cos(\omega_0 t - \frac{\pi}{2}) = \sin(\omega_0 t) \tag{14}$$

$$H\{\sin(\omega_0 t)\} = \sin(\omega_0 t - \frac{\pi}{2}) = -\cos(\omega_0 t) \tag{15}$$

where $\omega_0 > 0$

Hilbert transform in frequency domain:

$$TF\{\frac{1}{\pi t}\} = \begin{cases} -j, f > 0 \\ 0, f = 0 \\ j, f < 0 \end{cases} \tag{16}$$

In frequency domain, Hilbert transform is obtained by multiplying the values of Fourier transform at positive frequencies with $-j$ and at negative frequencies with $+j$ In fact, by multiplying by $\pm j$ the $\pm\pi/2$ phase shift it's realized.

$$H\{X(j\omega)\} = \hat{X}(j\omega) =$$
$$= -j\,sgn(\omega)X(j\omega) \tag{17}$$

Inverse Hilbert transform in frequency domain:

$$[\hat{X}(j\omega)]^{\wedge} = (-j\,sgn(\omega))^2 X(j\omega) =$$
$$= -sgn^2(\omega)X(j\omega) = -X(j\omega) \tag{18}$$

$$TF\{[\hat{X}(j\omega)]^{\wedge}\} = H\{H\{x(t)\}\} =$$
$$= -x(t) \tag{19}$$

Let there be $z(t) = x(t) + j\hat{x}(t)$ The Fourier transform of $z(t)$ is:

$$F\{z(t)\} = Z(j\omega) = X(j\omega) +$$
$$+ j(-j\,sgn(\omega)X(j\omega)) \tag{20}$$

$$Z(j\omega) = X(j\omega) + sgn(\omega)X(j\omega) \tag{21}$$

$$Z(j\omega) = X(j\omega)[1 + sgn(\omega)] \tag{22}$$

$$1 + sgn(\omega) = \begin{cases} 0, if \omega < 0 \\ 2, if \omega > 0 \end{cases} \tag{23}$$

Where:

$$Z(j\omega) = \begin{cases} 2X(j\omega)\sigma(\omega), \text{if } \omega > 0 \\ 0, \text{if } \omega \leq 0 \end{cases} \qquad (24)$$

It can be seen that the analytical signal has always a null spectral characteristic for negative frequencies. ([1]-[2], [9], [11]-[14])

3. Hilbert transform algorithm

To complete the Hilbert transform algorithm, it is necessary to follow the next steps:
1) Discretize the signal
2) Determine the Discrete Fourier transform
3) Multiply the obtained signal by $(\pm j)$
4) Apply the Invers Fourier transform

- *For example, it is given $x(t)$*

1) The discrete signal:
x[n] = {0;0.063;0.13;0.19;0.25;0.31;
0.37;0.43}
2) The Fourier transform:
X[k] = {1.7 + 0.0i; −0.3 + 0.6i; −0.2 + 0.2i; −0.2 +
0.1i; −0.2 + 00i; −0.2 − 0.1i; −0.2 − 0.2i; −0.3 − 0.6i}
3) The signal multiplied by $(\pm j)$:
$\hat{X}$[k] = {−0.7i; 0.6 + 0.3i; 0.2 + 0.2i; 0.1 + 0.2i;
0.2i; 0.1 − 0.2i; 0.2 − 0.2i; 0.6 − 0.3i}
4) The Inverse Fourier transform:
$\hat{x}$(t) = {0.23; −0.06; −0.06; −0.11; −0.11;
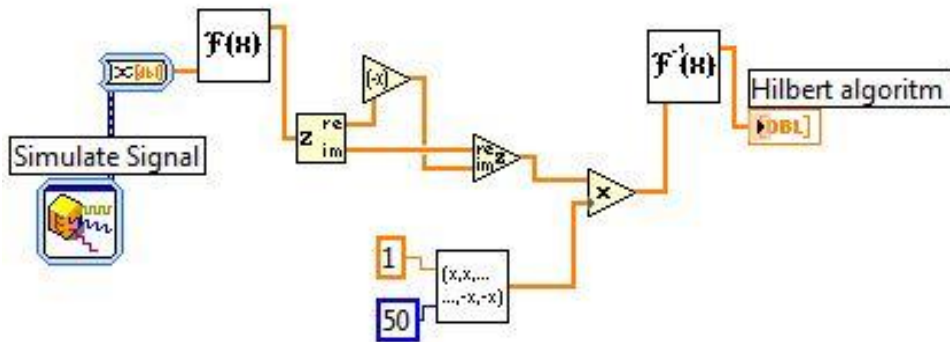−0.06; −0.06; 0.24}

Fig. 1: Virtual Instrument Block Diagram

4. Device Description

The implemented virtual instrument is a relative simple one and it can be observed in Figure 1.

To generate the signal, the program uses subroutine „Simulate Signal".

To determine Hilbert transform it is uses the existing subVI „Fast Hilbert Transform.vi".

To discretize the signal, the program uses subroutine „Convert from Dynamic Data"

To pass from time to frequency domain, the program uses subVI „FFT.vi":

To pass from frequency to time domain, the program uses subVI „Inverse FFT.vi".

To generate the discrete signal sir[n]={1,1,...-1,-1}, it is created a subVI, "Generate sir":
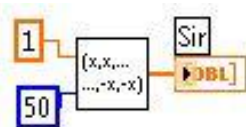

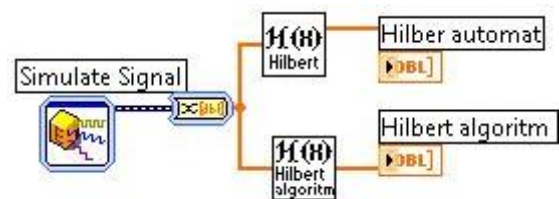Fig. 2: It is generated a signal with $\{\pm 1\}$ values.

5. Graphical

Fig. 3: Virtual Instrument Block Diagram to determine Hilbert transform by two different methods

- *For a sinusoidal signal input:*

As it can be seen in Figure 4 Hilbert transform is determined with two different methods:
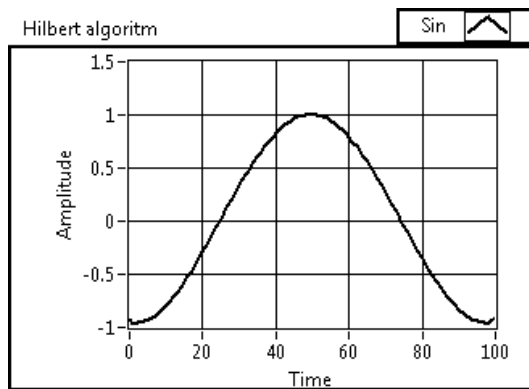1) the existing method in LabVIEW - „Fast Hilbert Transform.vi".
2) the algorithm created and implemented in LabVIEW - „Hilbert algoritm.vi".

Then, it was chosen a sine wave signal.

The Hilbert transform was determined with each method. The results were compared.
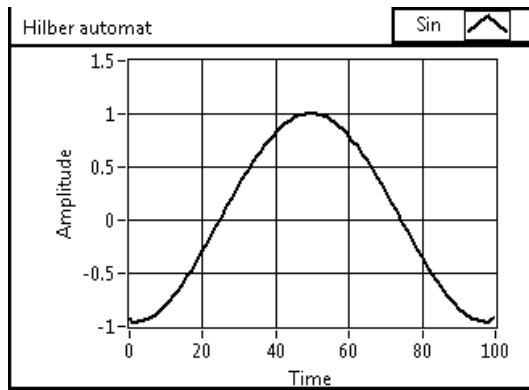
a)

b)

Fig. 4: Hilbert transform of the Sine wave signal input
a) with existing method   b) with implemented method

- *For a square wave input signal*:

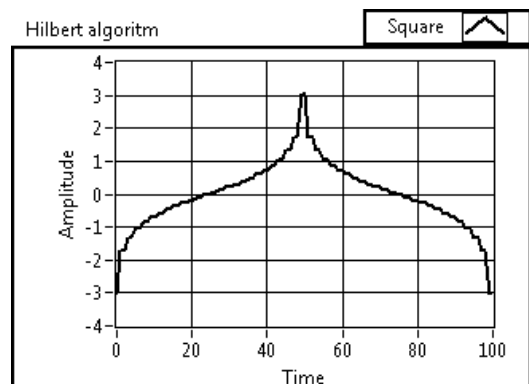$$H\{\textstyle\prod_{2a}(t)\} = \frac{1}{\pi}\ln\left|\frac{t+a}{t-a}\right| \qquad (25)$$

Fig. 5: Hilbert transform for a square wave input signal

- *For a triangle wave input signal:*

$$H\{1-|t/a|, |t|\le a|\} =$$

$$= -\frac{1}{\pi}\{\ln\left|\frac{t-a}{t+a}\right| + \frac{t}{a}\ln\left|\frac{t^2}{t^2-a^2}\right|\} \qquad (26)$$
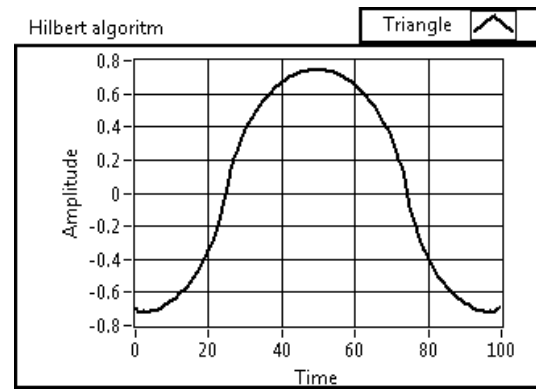

Fig. 6: Hilbert transform for a triangle wave input signal

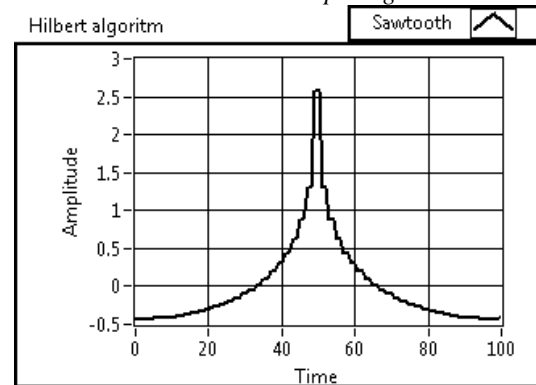- *For a sawtooth wave input signal:*

Fig.7: Hilbert transform for a saw tooth wave input signal ([8], [10], [15]-[19])

6. Conclusions

Time domain calculation of Hilbert transform was performed using polynomial approximations. This algorithm is difficult to implement.

The numerical method presented in this paper allows the passage form frequency domain to time domain, and vice versa, using the fast Fourier transform tool – tool which involves a small amount of computation and high computation speed.

This paper offers an alternative method for Hilbert transform calculation.

The algorithm obtained was implemented in LabVIEW software. The results were compared with the method proposed by LabVIEW environment.

The Hilbert transform can be used to create an analytic signal form a real signal.

As we seen it is possible to use the multiple Hilbert transform to calculate the inverse Hilbert transform.

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