



BUCKLING ANALYSIS OF INNOVATIVE CORRUGATED COLUMN BY USING RESPONSE SURFACE METHODOLOGY

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Abstract

Tubular steel structures with slender compression cross-sections are prone to local buckling. In this paper the axial compression is investigation of the innovative fabricated structural steel column consisting of sinusoidal corrugated pattern. When it comes to the design of compressive individuals, buckling is a crucial layout provision. The paper describes the effect of variation in the sectional properties like mean diameter, thickness, amplitude, and frequency of corrugation on the buckling axial load. A quadratic model was developed to correlate the independent variables for maximum buckling load by using central composite design (CCD) method. Subsequently ANOVA, a statistical tool is used to analyse and compare the different combinations and finding the most influential factor on buckling load. Response surface methodology is adopted to review the interaction models between the different combinations.

Key words: Innovative hollow column, Linear buckling analysis, ANSYS, Design of experiment, Response surface method

1. Introduction

(Innovative) In 1990's, Migita et al. [1] conducts 23 axial compressive tests on closed polygonal section steel columns. Some samples were stub columns with rectangular, pentagonal, hexagonal, heptagonal, and octagonal sections, whilst the others were medium-length columns with rectangular, hexagonal, and octagonal sections. Their research resulted in an empirical formula based on the compression tests to predict local buckling strength and interaction of closed polygonal steel columns between the local and total buckling capacities. An innovative X section was suggested by Chen and Jin [2] with intermediate stiffeners to improve the local buckling stress of thin-walled samples. The proposed

stub column was tested as hollow steel tubing as well as concrete-filled steel tubing.

From a material perspective, many construction materials, such as steel, aluminum, and the like, fall into the category of isotropic material. Moreover, such materials exhibit direction-dependent properties; thus, such materials are called anisotropic. The lateral deformations in other main directions may be smaller or greater in anisotropic materials stressed in one of the main directions than the deformation in the direction of the stress applied depending on the material properties [3]. The matrix of the material constants for a general anisotropic material includes 21 independent constants. It means all strains are linked to all stresses. This section involves some materials such as wood, plywood, and fibre-

reinforced plastics etc. These materials exhibit normal anisotropy. Besides plates made from anisotropic materials, due to their structure such as corrugated and stiffened plates, a number of plates also made of isotropic materials may also fall into the category of anisotropic plates. Such anisotropy form is called structural anisotropy [3].

Aoki and Ji [4] developed an innovative way of increasing the capacity of steel columns without sacrificing other properties. The aim was to weld steel tubes to square and triangular steel column apices. It was concluded that the plates provide lateral restraint to the steel tubes that postpone the buckling while the total load supported by the manufactured sections exceeded the load resulting from the summation of the capacity of the individual components.

Nassirnia et al. uses UHS (ultra-high strength steel) tubes that are welded to hollow corrugated column apices [5]. The performance of these sections is investigated under compression, both experimentally and numerically. Specific trapezoidal corrugation profiles are used to determine the impact of corrugation parameters on the columns proposed. Last, accost analysis is conducted to demonstrate the superior performance of innovative columns by comparing the fabrication cost of proposed sections with conventional sections fabricated from flat plates. Xiaolin et al. developed a theoretical solution for the mean crushing force of the

star-shaped tubes were derived, and the theoretical solutions show an excellent agreement with the experimental results [6].

This paper investigates the combined effect of mean diameter, amplitude, frequency and thickness on buckling load, and the process parameters were analysed using central composite design (CCD) in conjunction with RSM method.

2. Proposed corrugated column

Fig. 1 shows the hollow corrugated column with sinusoidal pattern used in the present paper. Since it is found that corrugation makes the structure more strength along the length of the column [7] as compared to conventional circular hollow columns. As buckling is the most important phenomenon as a result of axial-compression forces. Buckling analysis is used to determine buckling loads; the critical loads at which a structure becomes unstable, and the different ways that the structural member can deform which known as buckling mode shapes. In the present work, the corrugation parametric is analyzed with the range and set of combinations as shown in Table 2 and Table 3 by using design of experiment, in which Table 2 gives the details of all parameters in their coded and actual form and based on which the total number of simulation need to be performed is shown well in Table 3 with complete details keeping the length constant as 3048 mm.

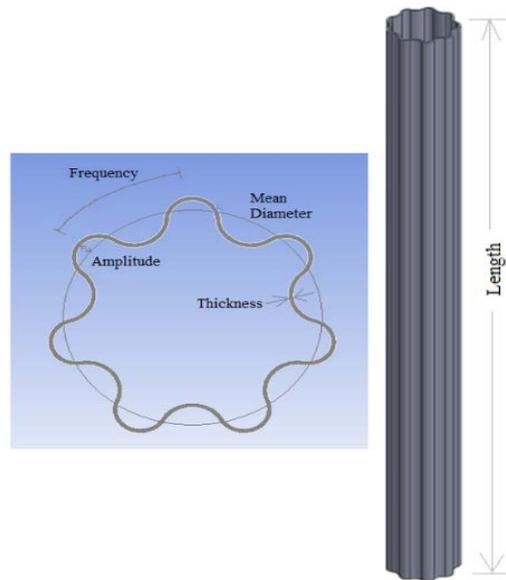


Fig. 1: A typical corrugated column

Material used is structural steel with specifications

- Young's Modulus $E=200\text{GPa}$
- Poisson's ratio $\nu=0.3$
- Density $\rho=7800\text{kg/m}^3$.

3. Multivariate design of experiment

The parameter used for buckling load by geometric parameters was analyzed by standard response surface methodology (RSM) design called central composite design (CCD). RSM is used for fitting a quadratic surface and also helps to optimize the process parameters with a minimum number of experiments and to evaluate the interaction between the parameters. [8]. RSM is a set of mathematical and statistical techniques that are useful in developing the empirical model construction, improving and optimizing process parameters and can also be used to evaluate the relationship between several factors that affect them [9]. RSM is a statistical approach that uses the quantitative data of the associated experiment to classify the regression model and optimize a response (output variable) affected by many independent variables. CCD is typically a $2n$ factorial run with $2n$ axial runs, and the experimental error is calculated by center runs (n_c). This experimental design consists of a factorial $2n$ with ± 1 notation coding increased by $2n$ axial points ($\pm a, 0, 0, \dots, 0$), $(0, \pm a, 0, \dots, 0)$, $(0, 0, \pm a, \dots, 0)$ and n_c center points $(0, 0, 0, \dots, 0)$ [10]. Each variable is investigated at two levels while as the number of variables (n) increases, the number of runs for a complete replication of the design is rapidly increasing. CCD was used for quadratic effect since $2n$ factorial designs cannot be calculated separately

for single second order effect. Hence, CCD has been used to establish the model for buckling load. The response with corresponding parameters was modelled in the statistical analysis to optimize the process conditions for the desired answer. ANOVA is used with the help of response surface methods to measure the statistical parameters.

In the assumed that the independent variable in the experiments are continuous, and experiments with negligible errors are controlled. The experimental design was aimed at maximizing the response variables (Y). An appropriate approximation to the true correlation between independent variables and response surfaces required to be found [10]. To reduce the error and effect of the uncontrolled factors, the experimental run was randomized. Using a second-degree polynomial equation Eq (1), the response was used to generate an empirical model correlating to the experimental variables.

$$Y = \beta_0 + \sum_{i=1}^n \beta_i X_i + \sum_{i=1}^n \beta_{ii} X_i^2 + \sum_{i=1}^n \sum_{j=i+1}^n \beta_{ij} X_i X_j + \varepsilon \quad (1)$$

where Y is the expected response; β_0 the constant coefficient; β_i the linear coefficients; β_{ii} the quadratic coefficients; β_{ij} the interaction coefficients; n the number of factors observed and optimized in the experiments; X_i and X_j the coded values of the variable leaching process parameters; and ε the random error. The codes are calculated as a function of the range of each factor as shown in Table 1 [11]. The test variables were coded in developing of the regression equation according to Eq. (2).

$$\chi_i = \frac{x - x_i^*}{\Delta x_i} \quad (2)$$

Table 1: Relationship between the coded and actual of the variables

Code	The actual level of variable
$-\alpha$	X_{\min}
-1	$[(X_{\max} + X_{\min})/2] - [(X_{\max} - X_{\min})/2\beta]$
0	$(X_{\max} + X_{\min})/2$
$+1$	$[(X_{\max} + X_{\min})/2] + [(X_{\max} - X_{\min})/2\beta]$
$+\alpha$	X_{\max}

Note: β is $2^{n/4}$; n is the number of variables (mean diameter, amplitude, frequency and thickness) and $n = 4$

Table 2: Independent variables and their coded levels for the central composite design

Independent variable	Symbol	Levels of coded variables				
		$-\alpha$	Low	Medium	High	$+\alpha$
		-2	-1	0	1	2
Mean Diameter	X ₁	228.6	247.65	266.7	285.75	304.8
Amplitude	X ₂	4.8	6.1	7.4	8.7	10
Thickness	X ₃	0.25	5.0625	9.875	14.68	19.5
Frequency	X ₄	5	7	9	11	13

Table 3: Experimental factors in coded and actual units and experimental responses

Standard run no.	Independent variable in coded form				Independent variable in actual form				Buckling load 'kN'
	X ₁	X ₂	X ₃	X ₄	Mean Diameter 'mm'	Thickness 'mm'	Amplitude 'mm'	Frequency	
1.	1	1	-1	-1	285.75	8.7	5.0625	7	3892
2.	-1	1	-1	1	247.65	8.7	5.0625	11	2593
3.	-1	-1	-1	1	247.65	6.1	5.0625	11	1875
4.	1	-1	1	1	285.75	6.1	14.6875	11	3561
5.	0	0	0	0	266.7	7.4	9.875	9	2975
6.	1	-1	-1	-1	285.75	6.1	5.0625	7	2796
7.	0	0	0	0	266.7	7.4	9.875	9	2975
8.	1	-1	1	-1	285.75	6.1	14.6875	7	3102
9.	-1	1	-1	-1	247.65	8.7	5.0625	7	2520
10.	1	1	-1	1	285.75	8.7	5.0625	11	3977
11.	1	1	1	-1	285.75	8.7	14.6875	7	4339
12.	1	-1	-1	1	285.75	6.1	5.0625	11	2861
13.	-1	-1	1	-1	247.65	6.1	14.6875	7	2101
14.	-1	-1	-1	-1	247.65	6.1	5.0625	7	1821
15.	0	0	0	0	266.7	7.4	9.875	9	2975
16.	0	0	0	0	266.7	7.4	9.875	9	2975
17.	-1	-1	1	1	247.65	6.1	14.6875	11	2491
18.	-1	1	1	1	247.65	8.7	14.6875	11	3456
19.	1	1	1	1	285.75	8.7	14.6875	11	4984
20.	-1	1	1	-1	247.65	8.7	14.6875	7	2914
21.	0	0	0	-2	266.7	7.4	9.875	5	2768
22.	0	0	0	2	266.7	7.4	9.875	13	3276
23.	2	0	0	0	304.8	7.4	9.875	9	4350
24.	0	0	0	0	266.7	7.4	9.875	9	2975
25.	0	0	0	0	266.7	7.4	9.875	9	2975
26.	0	2	0	0	266.7	10	9.875	9	3910
27.	-2	0	0	0	228.6	7.4	9.875	9	1917
28.	0	-2	0	0	266.7	4.8	9.875	9	1975
29.	0	0	-2	0	266.7	7.4	0.25	9	2673
30.	0	0	2	0	266.7	7.4	19.5	9	3759

where χ_i is the i^{th} independent variable of the dimensionless coded value; the uncoded value of the i^{th} independent variable is denoted by X_i ; similarly, at the center point, the uncoded value of the i^{th} independent variable denoted by X_i ; and the step change value has been defined by DX_i [12].

N as a total number of runs or experiments means the number of tests required for the CCD, the usual $2n$ factorial points with their root in the middle. The quadratic terms generate from the center at a distance a (a value was set at 2 in this study), from a set $2n$ axial points, and run the replicates at the center to prevent error; where n is the number of independent variables; The axial points ($2n$) are for screening analysis and readability, which tests the variance of model estimation and is constant from the design core at all points equidistant and n_c is the number of central points that reproduce the test at the core and is very important for the independent evaluation of the experimental error. [13].

Cube points:	16
Center points in cube:	4
Axial points:	8
Center points in axial:	2

$$N = 2n + 2n + n_c \quad (3)$$

In the CCD design, eight factorial points, six axial points, and six replicates were used at the central points in four variables in the experiments. The total number of tests (N) needed for the four independent variables thus amounts to 30 as shown in Eq. (3). Upon description of the desired range value of the variables, they are coded to ± 1 for factor points, 0 for center points and $\pm \alpha$ for factor for the axial points.

4. Numerical Modelling

The linear buckling behavior was analyzed using ANSYS workbench software 18.1. The tubes are meshed with the element size of 2mm and condition of constraints are: one end is fixed while load is applied on the free end.

In ANSYS, buckling analysis might be conducted using Eigenvalue buckling analysis [14]. Eigenvalue buckling analysis reveals the buckling load factor that has to be multiplied by the applied loads to reach buckling point [15]. Eigenvalue buckling analysis may be based on linear or nonlinear static analysis. Linear eigenvalue buckling analysis deals with ideal elastic structure and provides a non-conservative estimation that is useful for later complicated nonlinear buckling analyses. ANSYS provides a ready-to-use tool for eigenvalue

buckling analysis and calculates buckling load factors along with buckling mode shapes.

5. Model fitting and statistical analysis

ANOVA (Analysis of variance) has been used for graphical analysis of the data to define the interaction between the process variables and the responses to estimate the statistical parameters.

The statistical tool Design Expert, Stat-Ease, Inc., Minneapolis, USA, was used to analyze experimental data for regression, plot the response surfaces, and plot contour at optimized level. The statistical significance within the same system was tested by the F-test. The exactness of the equipped polynomial model has been calculated by the R_2 coefficient. The important model terms were evaluated by the probability value (P-value) at 95% confidence interval.

6. Results and discussion

6.1. Development of model

The statistical ‘‘Design-Expert Version 12’’ software has been used to study the regression analysis of result data and to get the response surface plot. The statistical parameters were examined by using ANOVA. For the buckling load study, the required geometric parameters range and coded level of variables are given in Table 3 and 4, which shows the design of experiments together with the simulation results. As the output proposed by the Design-Expert software, the quadratic model is suggested. The final empirical model in terms of a coded factor for buckling load is shown in Eq. (4):

$$\text{Buckling load (kN)} = 2974.80 + 1217.20 X_1 + 994.65 X_3 + 565.47 X_2 + 277.47 X_4 + 419.55 X_1 X_3 + 77.00 X_1 X_2 + 48.80 X_1 X_4 + 202.25 X_3 X_2 + 94.25 X_3 X_4 + 439.00 X_2 X_4 + 159.68 X_1^2 - 31.22 X_3^2 + 242.38 X_2^2 + 48.18 X_4^2 \quad (4)$$

where the negative sign indicates the antagonistic effects, whereas the positive sign indicates the synergistic effects.

So as to fit a good model for significance of the regression model and individual model coefficients with lack of fit test was performed. The significant factors were typically rated with a confidence level of 95% based on the F-value or P-value (probability value). Table 4 shows the ANOVA for the data generated by Eq. (4) for buckling load. The larger value of F and the smaller value of P (Prob. > F) shows more significant of the corresponding coefficient [16]. The F-value of 1462.72 indicates that the model is significant. Moreover, the model terms are significant only when the values of ‘‘Prob. > F’’ are less than 0.05.

Table 4: Analysis of variance (ANOVA) for response surface quadratic model for buckling load

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	1.777E+07	14	1.269E+06	1462.72	< 0.0001	significant
X ₁ -Mean Diameter	8.889E+06	1	8.889E+06	10242.64	< 0.0001	significant
X ₂ -Amplitude	1.919E+06	1	1.919E+06	2210.56	< 0.0001	significant
X ₃ -Thickness	5.936E+06	1	5.936E+06	6839.57	< 0.0001	significant
X ₄ -Frequency	4.619E+05	1	4.619E+05	532.24	< 0.0001	significant
X ₁ X ₃	1.760E+05	1	1.760E+05	202.82	< 0.0001	significant
X ₁ X ₂	5929.00	1	5929.00	6.83	0.0196	
X ₁ X ₄	2381.44	1	2381.44	2.74	0.1184	
X ₃ X ₂	40905.06	1	40905.06	47.13	< 0.0001	significant
X ₃ X ₄	8883.06	1	8883.06	10.24	0.0060	
X ₂ X ₄	1.927E+05	1	1.927E+05	222.06	< 0.0001	significant
X ₁ ²	43712.17	1	43712.17	50.37	< 0.0001	significant
X ₃ ²	1670.54	1	1670.54	1.92	0.1856	
X ₂ ²	1.007E+05	1	1.007E+05	116.04	< 0.0001	significant
X ₄ ²	3979.94	1	3979.94	4.59	0.0491	
Residual	13018.31	15	867.89			
Lack of Fit	13018.31	10	1301.83			
Pure Error	0.0000	5	0.0000			
Cor Total	1.779E+07	29				

Level of every factor was determined dependent on primer expulsion. The distance of the axial points from the center was ± 2 and it was calculated from $a = (2n)^{0.25}$ where n is the number of variables. The signal-to - noise ratio is determined by accuracy, consisting of the expected value at the design points and the average predictive error. In the present study, adequacy precision ratio is 147.2947 and is desirable due to this ratio has been greater than 4. Hence, the developed model can be used to guide the design space. The adequacy of the

developed model was the main part to check the data analysis of the experiment. The normal probability and studentized residual plot were shown in Fig. 2 and it has been found that there was neither response transformation nor any apparent problem with normality. This signifies that for all values of the response, the random scatter plot, the variance of original observation is constant, and this was an indication that there was no need for transformation of response variables.

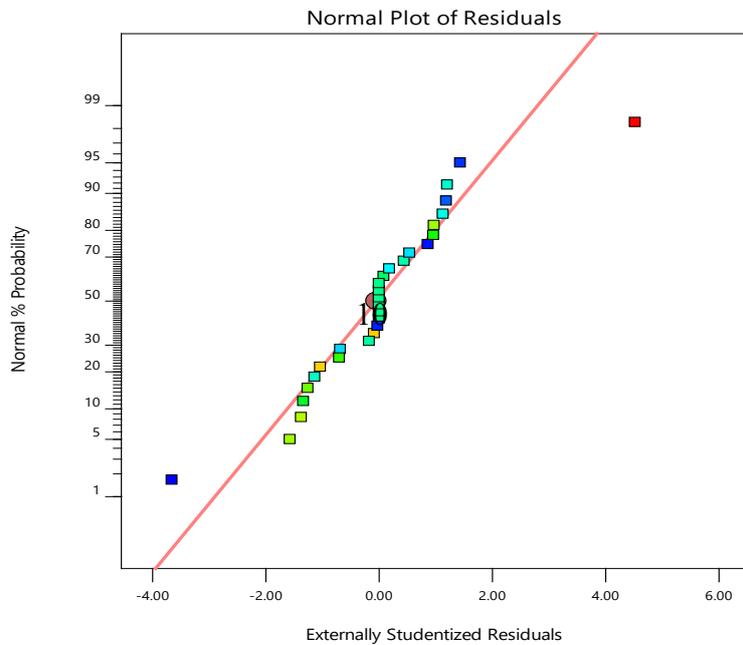


Fig. 2: External studentized residuals and normal percentage probability plot for buckling load

Actual and the predicted percentage for buckling load was shown in Fig. 3.

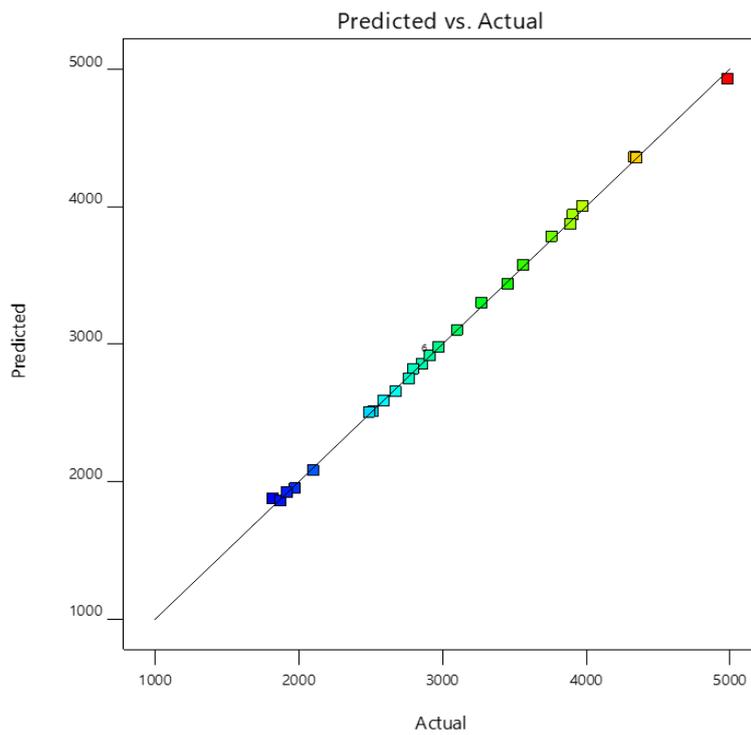


Fig. 3: Actual and predicted plot of buckling load (kN)

It was found that the values of R_2 and adjusted R_2 (R_{adj}) were 99% and 99% respectively. The value of R_2 describes up to what extent perfectly model can estimate the numerical data points and the

adjusted R_2 measured the amount of variation about mean explained by the model. The predicted R_2 value was 0.9958, which was near close to R_2 value. It was revealed that the numerical data for buckling

load is fitted well with the predicted value of the model. The standard deviation for the model was 29.46. The smaller value of standard deviation shows good model that gives near value between

predicted and actual values for the responses. The statistical parameter obtained from ANOVA was shown in Table 5.

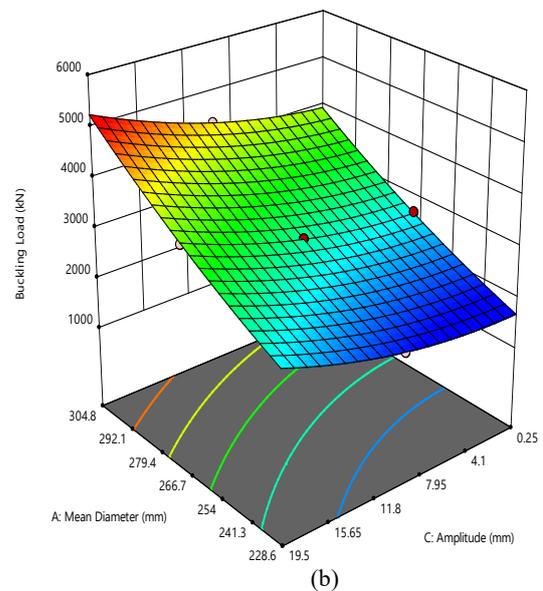
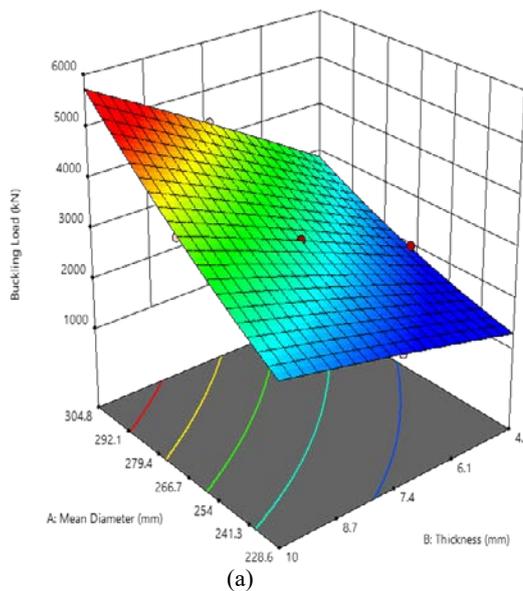
Table 5: Statistical parameters obtained from the analysis of variance (ANOVA) for the models for buckling load

Insignificant factors excluded	Buckling load 'kN'
Standard deviation (S.D.)	29.46
Mean	3058.61
Coefficient of variance (C.V) %	0.9632
Adeq. precision	147.2947
R ₂	0.9993
Adjusted R ₂	0.9986
Predicted R ₂	0.9958

6.2 Combined effect of sectional parameters on buckling load

Response surface methodology was used to investigate the individual and interaction effect of the four-factor on buckling load. Based on ANOVA, the results were obtained, the effects of geometric factors on buckling load, corresponding three-dimensional response surface plots were shown in Fig. 4–9 and the response model was represented in Eq. (4). Fig. 3 show the relationship between the actual and predicted values of buckling load. The mean diameter, thickness, amplitude and frequency have significant effects on the buckling load. With the response surface analysis method, it was found

from Table 3 that geometric factors influence the buckling load. It has been found from Table 4, mean diameter (X_1) was found to be the major effect on the buckling in comparison to other variables as also found in the major researches, which may be due to the high F value of 10242.64 for mean diameter, whereas the parameter frequency has been found to have less significant effects for buckling load. However, the interactions between the variables have no significant effect on buckling load. But on considering the main corrugation effect amplitude is found to be more effective as compared to frequency.



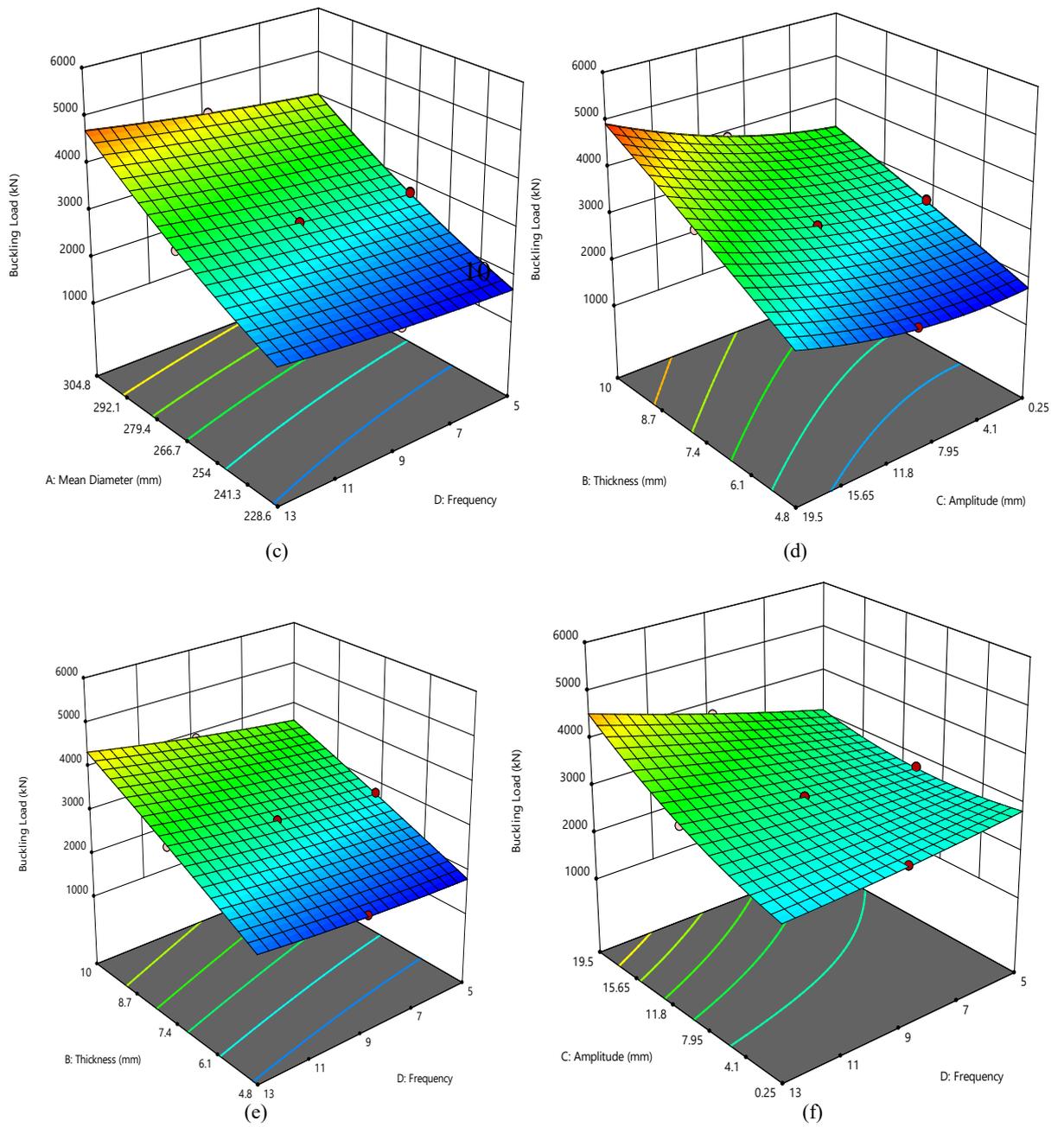


Fig 4: Response surface graph for the proposed sinusoidal corrugated tube for buckling load force; (a) effect of mean diameter and thickness (b) effect of mean diameter and amplitude, (c) effect of mean diameter and frequency, (d) effect of thickness and amplitude, (e) effect of thickness and frequency, (f) effect of amplitude and frequency.

7. Conclusion

The main purpose of the investigation presented in this paper was to find out the buckling load of the corrugated geometry using Ansys Workbench is carried out. To get the standard number of experiments need to be perform to have all possible combinations results, design of experiment is applied with central composite design technique. Finally, to have a clear understanding about the effect of each geometric parameters over the buckling load response surface method is used. The regression analysis is calculated by using design expert for predicting the response behavior. The statistical significance, regression analysis and response surface analysis were carried out using the buckling results obtained values at variable operating conditions. A model was formulated to correlate the buckling load values with the variables to the responses. Based on models, the response surfaces behaviors are shown in Fig. 4 (a, b, c, d, e & f) with two parameters effects over the buckling load and makes the influence effect much better to understand. Out of the corrugated parameters apart from mean diameter and thickness the amplitude have played an important role over the buckling load as compared to frequency. The kind of the structures are still under study and development as it is called as innovative structures.

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