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MATHEMATICAL MODELS USED IN THE OPTIMIZATIONS OF SUPPLY CHAINS

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Abstract

This paper provides a synthesis of the concept of supply chain optimization. The aim of the research is to identify a supply chain optimization process, the key aspects in carrying out this process, and at the same time to emphasize the advantages that the optimization of companies' processes brings through the mathematical modeling of logistic processes that can be useful in the optimization of supply chains. In the research methodology it is used the Simplex transport algorithm. In a practical situation of a company with 3 warehouses and 4 clients, it allowed optimizing the transport routes and getting a minimal total cost for logistical delivery.

Key words: supply chain management, optimization, mathematical modeling, transportation, risk management

1. Introduction

The supply chain is a system of organizations, people, technology, activities, information and resources involved in moving a product or service from supplier to customer. Supply chain activities transform natural resources, raw materials and components into a finished product, which is then delivered to the final customer. The supply chain incorporates the production process along with the purchase of raw materials and the distribution of finished products, encompassing all the existing process responsible for the transformation of materials taken from the supplier to the finished products delivered to customers [1].The supply chain can also be defined as a chain of demand satisfaction by several companies or as a value chain [2]. Supply chain management includes the planning and management of all activities involved in the process of procuring and receiving from the source, conversion, and management of all logistics activities. This includes coordinating and collaborating with partners on the same communication channel, which may be suppliers, intermediaries, third party service providers and customers. Essentially, supply chain management integrates supply and demand management within and between companies. This includes all logistics management activities mentioned above, as well as manufacturing operations, but also directing process coordination and marketing, sales, product design, finance and information technology activities [3].

The management of logistics activities can also be defined as the set of strategic management processes for the acquisition, movement and storage of raw materials, semi-finished products and finished products, within the enterprise and the supply and distribution channels, with the aim of satisfying orders with the lowest costs for the enterprise and the creation of added value [4]. Due to its global nature and systemic impact on a firm's financial performance, the supply chain arguably faces more risks than other areas of the firm. Risk is a fact of life for any supply chain, whether it is related to quality and safety, material shortages in production, legal issues, security, regulatory and environmental compliance issues, natural disasters, or terrorism - there is always an element of risk [5]. Within organizations, risks can arise under different aspects: market evolution, project failures, accidents, natural disasters, etc. The importance of proper risk assessment and prioritization is significant [6]. Implementing supply chain management in the industry can help reduce costs, increase company profits in supply management and delivery planning management [7]. On the other hand, the importance of supply chain management is also given by the aspect of sustainability, as the resources available in our world are decreasing, and the effects of climate change and environmental degradation are increasing [8]. Supply chain risk management is the systematic identification, assessment and quantification of possible chain disruptions, with the objective of controlling risk exposure and reducing its negative impact on achieved performance [9].

According to a study by Giuffrida et al. [10], the risk management strategies adopted depend on the type of uncertainty faced by companies at the logistics level and, to a lesser extent, on the industry in which the company operates. Also, other factors such as country of origin, company size or company type do not exert any significant influence. According to estimates, the global logistics market is expected to grow to 12.68 billion dollars by 2023, but the COVID-19 pandemic has shown us how an unforeseen event can reveal the vulnerability of supply chains and implicitly, the inefficiency of logistics. Logistics is a critical part that connects different supply chain players and greatly influences overall resource utilization and business performance [11]. Due to the growing and more frequent problems that have appeared recently in all the supply chains on the world map, the specialists considered it appropriate to implement and develop mathematical models that would lead to the minimization of these problems and consequently to the efficiency of the supply chains. An analysis of these models over the last two decades was realized in a study carried out by Habib et al. [12]. A more recent analysis of the specialized literature is proposed by Nguyen et al.[13]in a paper in which the collected mathematical models are based on are mostly based on linear and mixed programming based on integers and which use variants of supply chain optimization stochastic programming and robust optimization.

The objective of this article is to demonstrate the effectiveness of mathematical models in the optimization of supply chains with the support of an algorithm adequate for modeling of logistics processes.

2. Material and method

The methodology of the study consisted in identifying the opinions of specialists regarding the supply chain optimization process, the key aspects in carrying out this process, and at the same time to emphasize the advantages that optimization of companies' processes brings through the mathematical modeling of logistics processes.

In this sense, it was employed the Simplex transport algorithm. This is useful in optimizing transport algorithm which optimizes the routes so that the total cost is minimal.

In order to organize transport as efficiently as possible, we need to know both what all possible transport routes are and what the cost of each route is. Identifying the best route involves finding the optimal combination between the quantity transported from each warehouse and its recipient, with the minimum transportation cost.

The Simplex transport algorithm models an economic situation in which "*n*" consumers (beneficiaries) are supplied from m centers (sources, warehouses).

Solving a transport problem using the Simplex algorithm involves going through two stages:

1. Finding an initial feasible solution using the Northwest corner method or the unit cost method
2. Finding the optimal solution using the modified distribution method (MODI) and Stepping Stone methods.

3. Results

We started from the idea that a company that has 3 warehouses (D1, D2 and D3) needs to send products to 4 customers (C1, C2, C3 and C4). The quantities available in each warehouse, the quantities requested by each customer, as well as the unit transport costs, expressed in m.u. (monetary units), are known (written in the upper right corner of each cell of the transport table – Table 1).

Table 1: Initial data for transport problem

	C1	C2	C3	C4	**Available quantity**
D1	3	2	7	6	**5000**
D2	7	5	2	3	**6000**
D3	2	5	4	5	**2500**
Requested quantity	**6000**	**4000**	**2000**	**1500**	

In the first step of solving this problem we will check if the problem is balanced or not. A problem is balanced if the sum of the quantities available in warehouses equals the sum of the quantities demanded by customers.

Doing the calculations we find that the sum of the available quantity (5000+6000+2500) is equal to the sum of the requested quantities (6000+4000+2000+1500), respectively equal to 13,500 m.u.

The next step is to identify an initial solution, which we can obtain as we said before, either with the

N-W corner method or with the minimum cost method.

The N-W corner method involves the following steps:

1. The Northwest corner of the transportation table is identified.

2. The route related to the one located in the N-W corner of the transportation table is assigned a maximum flow determined as the minimum of the quantity available in the warehouse located on the respective route and the quantity requested by the beneficiary located on the same route.

3. The two quantities (the available quantity and the requested quantity) are reduced with the flow allocated to the cell and the line and/or column which, following the allocation remained with a zero quantity are removed from the transport table.

4. The previous step is repeated until all available quantities are allocated.

The resulting solution is a feasible solution (a solution that satisfies the requirements of the problem). The transport effort (total transport cost) related to the obtained solution is calculated.

If we apply this method, we get the solution presented in Table 2.

Table 2: Initial solution found with the N-W corner method

	C1	C2	C3	C4	Available quantity
D1	3 5000	2	7	6	**5000**
D2	7 1000	5 4000	2 1000	3	**6000**
D3	2	5	4 1000	5 1500	**2500**
Requested quantity	**6000**	**4000**	**2000**	**1500**	**13.500**

As can be seen, customer 1 will receive 5000 products from warehouse 1 and 1000 products from warehouse 2. Customer 2 will receive the entire requested quantity of 4000 pieces from warehouse D2. Customer 3 will be supplied with 1000 pieces each from warehouses D2 and D3. Customer 4 will receive the goods from warehouse 3.

This solution assumes a total cost of 55,500 m.u., obtained by summing the products between the allocated quantity and the unit transport cost, for each occupied cell in the Table 2:

(5000x3 + 1000x7 + 4000x5 + 1000x2 + 1000x4 + 1500x5).

A cell is considered occupied if a certain quantity is transported on the route corresponding to the cell.

The second option for identifying the initial solution was the minimum unit cost method. To apply this method, the cell or cells for which the unit transport cost is minimal is identified. If there are many such cells, start with the one that allows the highest maximum flow. The cell thus found is assigned the maximum flow determined as the minimum between the quantity available at the supplier on the respective routes and the quantity requested by the beneficiary on the same route. Decrease the available quantity and the requested quantity with the allocated flow, eliminating the row and/or column for which the quantity became zero after the allocation.

The step is repeated until all available quantities are allocated. The total transport effort related to the obtained feasible solution is calculated.

The result obtained using this method is presented in Table 3.

Table 3: Initial solution found with minimum unit cost method

	C1	C2	C3	C4	Available quantity
D1	3 1000	2 4000	7	6	**5000**
D2	7 2500	5	2 2000	3 1500	**6000**
D3	2 2500	5	4	5	**2500**
Requested quantity	**6000**	**4000**	**2000**	**1500**	**13.500** **13.500**

The identified solution assumes that the first customer receives 1000 products from the first warehouse and 2500 products each from warehouse 2 and 3. Customer 2 will receive the entire requested quantity of 4000 pieces from warehouse D1. Customer 3 and customer 4 will be supplied with the entire quantity of 2000 pieces and 1500 pieces respectively from warehouse D2.The total cost in this situation is lower than the previous one, being 42,000 m.u. which is computed as follows:

(1000x3 + 4000x2 + 2500x7 + 2000x2 + 1500x3 + 2500x2).

It shows that by the second method we found a better solution than the original one. The next step would be to test whether the second solution is also the optimal solution of this method. For this we first turn to the MODI method. We check whether or not the initial solution is non-degenerate. A solution is non-degenerate if it checks the condition:

$$m+n-1= \text{no. of occupied cells} \quad (1)$$

where m=no. of deposits and n=no. of beneficiaries.

In our case we have 3 deposits and 4 beneficiaries, which lead to obtaining the result 6 and correspond to the number of occupied cells in the obtained solution. For the occupied cells we will solve the system of equations of the form:

$$f_i+d_j=c_{ij} \quad (2)$$

where: f_i – are variables associated with each deposit, d_j - are variables associated with each beneficiary, c_{ij} - is the unit cost of transport from warehouse i to beneficiary j.

The system of equations is solved by giving an arbitrary value to one of the unknowns. For the free cells we will solve the system of equations of the form:

$$e_{ij} = c_{ij}-f_i-d_j \quad (3)$$

If all values are positive or equal to zero, the verified solution is the optimal solution. Otherwise, the cell for which e_{ij} has the lowest value among the negative values is chosen (if there are several cells with the same value, any one is chosen), that cell being called the entry cell in the transport table. The variable e_{ij} has the meaning of economy in unit transport costs and tells us what economy will be achieved in the next solution if the route related to that cell is used.

Following the calculations, we found that the tested solution is not the optimal one, obtaining a negative value of the economy at unit transport costs, at the intersection of warehouse 2 with customer 2. This cell is considered the input cell for the Stepping Stone method, which will identify the output cell in the transport table. For this, the transportation table is assimilated to a pool of water and the occupied cells to stones that protrude above the surface of the water (and can be stepped on).Starting from the entry cell in the transportation table, a route is searched that returns to the point of departure, respecting the following rules:

1. Movement is only horizontal or vertical
2. At each step only on stones or in the entrance cell.
3. After each step, the direction of travel is changed by 90 degrees to the left or right.

Note: At each step one can jump over other rocks or over other water surfaces.

Table 4: Stepping Stone method - water pool

	C1	C2	C3	C4	Available quantity
D1	(+) 3; 1000	(−) 2; 4000	7	6	5000
D2	(−) 7; 2500	(+) 5	2; 2000	3; 1500	6000
D3	2; 2500	5	4	5	2500
Requested quantity	6000	4000	2000	1500	13.500 / 13.500

input cell

The entry cell is marked with the + sign and alternatively with the -, +, - signs, the other cells on the previously found route.

The minimum of the flows of the cells marked with the - sign is calculated, and with the obtained value proceed as follows: the flows of the cells marked with the - sign are decreased by the determined value and the flows of the cells marked with the + sign are increased by the same value.

Following this operation one or more cells will have flow 0; those cells are called output cells in the transport table. We have obtained the solution to the problem as presented in table 5.

Table 5: Optimal solution using the Stepping Stone method

	C1	C2	C3	C4	Available quantity
D1	3; 3500	2; 1500	7	6	5000
D2	7	5; 2500	2; 2000	3; 1500	6000
D3	2; 2500	5	4	5	2500
Requested quantity	6000	4000	2000	1500	13.500 / 13.500

For this solution, the total cost is 39,500 u.m., which is computed as follows:

(3500x3+ 1500x2+ 2500x5+ 2000x2+ 1500x3+ 2500x2).

It is the lowest value obtained so far, which implies an economy of 16,000.m u. compared to the first solution. So, the solution to the analyzed problem assumes that the first customer receives 3500 products from warehouse D1 and 2500 products from warehouse D3, the second customer receives 1500 and 2500 products respectively from warehouses D1 and D2, and customers C3 and C4 to receive the entire requested quantity (respectively 2000 and 1500 products) from warehouse D2.

4. Conclusion

Traditionally, supply chains are viewed from the perspective of several organizations working together, from suppliers to final consumers, or, it is associated with different types of flows (e.g. material or financial) belonging to certain processes, such as procurement, production and distribution. Implementing a sustainable supply chain is not something that can be done overnight. This must be part of a genuine desire for real change, change that must first come from a company's executives and then be enacted by every member of every company within a supply chain.

According to a study by several Chinese researchers[14],the implementation of more effective risk management programs in supply chains, based on advanced artificial intelligence technologies such as machine learning (ML), can provide much better future predictions. ML algorithms perform well in identifying anomalous risk factors and deriving predictive insights from historical datasets. Another point of view is offered by Maicom and Moacir [15], who claim that there is a tendency in some companies to consider the existence of not one, but several supply chains within the organization.

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